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Essays on Heterogeneous Firms and Corporate Default

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"Research is like sausages: the finished article can be delicious but people don't want to see what goes into the making of it"
Clarendon Lectures, John H. Moore

…and this section is about those that played an important role in the making of this manuscript.

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Abstract

The recent US and European financial crises have witnessed the demise of a multitude of firms in both continents, with the headlines of the newspaper filled up by the default experience of several key industrial realities. Both in the US and Europe, the policy authorities have been obligated to re-think in depth the design of their bankruptcy laws. Despite the top priority of the matter in hand, the policy makers have lacked a consistent framework able to *quantify* the economic impact of the bankruptcy reforms. My research enquiry enters here. In the three chapters of my thesis I investigate the phenomenon of corporate default, with a particular focus on the macroeconomic consequence of changes in the corporate bankruptcy law.

Chapter 1 studies the general equilibrium implications of changes in the corporate bankruptcy law. I address this question by building and characterizing a general equilibrium firm dynamics model in which the default option replicates salient features of the U.S. Bankruptcy Code: distressed firms can voluntarily file either for liquidation (Chapter 7) or reorganization (Chapter 11). The model is consistent with several regularities on corporate bankruptcy and firm dynamics. I find that changes in the bankruptcy law design have economically significant general equilibrium effects on output, consumption and TFP, through firms' selection.

Chapter 2 documents a novel channel through which pro-creditor bankruptcy reforms can backfire. The theory arises from the observation of the fact that firms file for bankruptcy reorganization (Chapter 11) not only to restructure debt but also to restructure labour contracts. When workers extract rents, restructuring labour contracts helps distressed firms to regain economic soundness. Shareholders weigh the cost of restructuring labour contracts against their claims on the value of the firm. In this environment, bankruptcy reforms face a trade-off. A more creditor-friendly law raises recovery values of successful reorganizations. Yet, it reduces shareholders' claims and discourages the restructuring of labour contracts: reorganizations are more likely to fail and firms get liquidated. As a result, pro-creditors reforms can cause expected recovery values to fall and raise the cost of debt. I characterize this trade-off in a static model and show that the optimal level of creditor rights decreases with the bargaining power of workers. To test the theory, I exploit the heterogeneity in the U.S. states unionization coverage, and a shift towards a more creditor-friendly Chapter 11 in 1998. I then develop a firm dynamic model and calibrate it to the pre-1998 period. The model can account for the larger fall in the relative use and likelihood of success of Chapter 11 in regions where workers extract more rents.

Chapter 3 (joint work with Omar Rachedi) studies the series of US annual corporate default rates from 1950 until 2012. We document the presence of one structural break in the unconditional mean, which is dated in 1986. Meanwhile credit spreads hardly moved.

We present a dynamic equilibrium model where the development of credit markets accounts for this empirical evidence. Financial development increases both the default rate and firms' expected recovery rates. These two effects offset each other and translate into constant credit spreads. In the model financial development explains 64% of the rise in default rates and predicts just a 2 basis point increase in the credit spreads. Furthermore, the model accounts for a number of trends that characterized public firms over the last decades: the fall in the number of firms distributing dividends, the rise in the degree of dividend smoothing, and the increase in the volatility of public firms.

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CHAPTER 1

A Quantitative Theory of Corporate Bankruptcy

1.1 Introduction

What are the general equilibrium implications of changes in the corporate bankruptcy law design[1]? To answer this question I build a general equilibrium firm dynamics model[2] in which the default option replicates salient features of the U.S. court-supervised[3] bankruptcy procedures: distressed firms can *voluntarily*[4] file either for liquidation (Chapter 7, Ch 7) or reorganization (Chapter 11, Ch 11). In liquidation (cash auction procedures) the bankruptcy court appoints a trustee to shut the firm down, sell its assets in a cash-auction, distribute the proceeds to the creditors and dissolve the corporation. In reorganisation (negotiation procedures) debtors and creditors agree upon a restructuring plan that allows the firm to recover its financial soundness and the creditors to be partially refunded.

To discipline the model, I collect facts that relates firms characteristics with filing decisions and creditors recovery values (Bris et al. [2006]). All else equal: small firms tend to file for liquidation (Ch 7), while big firms tend to file for reorganisation (Ch 11); more leveraged firms tend to file for reorganisation (Ch 11); creditors' losses are lower in reorganization (Ch 11) than in liquidation (Ch 7); creditors' losses increase in leverage. Ulti-

[1]The bankruptcy law design represents the legal ways by which firms can repudiate on their debt obligations.

[2]This paper adds to the vast literature on firm dynamics pioneered by Hopenhayn [1992], in which the author extends the long run industry equilibrium theory by introducing a concept of stationary equilibrium which allows him to investigate the phenomena of entry, exit and heterogeneity in the size and growth rates of firms.

[3]The analysis abstracts from out-of-court bankruptcy workout and pre-packaged bankruptcy (which combines features of an out-of- court restructuring with some of the features of a traditional Chapter 11).

[4]This analysis neglects involuntary bankruptcy petitions governed by Section §303 of the Bankruptcy Code. Nonetheless, the prospect of creditor liability for costs, attorney's fees, damages, and possibly punitive damages makes involuntary petitions one of the lesser-used creditor tools. In particular, involuntary bankruptcy is most often used when unsecured creditors suspect fraud on the part of a company, such as when a Ponzi scheme is discovered, or for some other extraordinary reason. Otherwise, creditors will typically pursue collection of their own claims directly, including through litigation in state or federal court. That might end up forcing the company into bankruptcy, but technically it would be a bankruptcy of the voluntary kind. In the data set considered in this paper, 161 out of the 166 files are 'voluntarily' bankruptcy petitions.

mately, I document that recovery values under Ch 11 are independent on the initial level of debt (using the UCLA LoPucki Bankruptcy Research Database).

In the theoretical analysis, I show that the model can analytically account for some facts. I then calibrate the model to the US economy from 1979-2012, and show that the model can quantitatively account for the remaining facts. To do so, I merge firms' level accounting data from Compustat North-America Fundamentals Annual with bankruptcy information from the UCLA LoPucki Bankruptcy Research Database.

The main gearwheel of the mechanism is the selection of firms. Changes in the corporate bankruptcy law design *directly* affect the cost of the debt service and therefore profitability. By doing so, they *indirectly* affect firms decision to exit, liquidate or reorganize, triggering selection. From this point of view, a novel contribution of the paper is to investigate the effects of changes in the bankruptcy law design on the *cross section distribution* of firms.

In the quantitative analysis, I perform four counterfactuals to study the general equilibrium implications of changes in the corporate bankruptcy law design.

First, I investigate what would have happened if Ch 11 had never been introduced in the 1978 reform of the Corporate Bankruptcy Code (Section 1.8.1). Results are striking: median TFP increases by 1.44%, net output falls by 1% (similarly consumption, by 0.7%).

Second, I quantify the relevance of a particular feature of Ch 11: the debtor in possession financing - the right to borrow during reorganization (Section 1.8.2). The practice of debtor-in possession (DIP) financing boomed with the development of the junk bond market during the 1990s (Miller [2007]), but it was hit drammatically by the onset of the financial crises, when the high-yield bond market froze (Martin et al. [2009]). To study the general equilibrium implications of this event, I compare the baseline economy with one where firms cannot borrow during reorganization. Ch 11 becomes less attractive (Ch 11 defaulters drop by 70%). Default becomes more costly: median Ch 11 recovery values drop by 34%, and the equilibrium average interest rates increase by 6 basis points. Despite TFP does not vary much (drop of 0.13%), output and consumption fall significantly ($\sim$ 0.5%) due to the churning of large productive firms.

In the last two exercises I attack two of the main organizing questions of the last two decades of corporate bankruptcy literature: Do we need Ch 11 when Ch 7 is efficient (Section 1.8.3)? What is the optimal level of creditor rights (Section 1.8.4)?

The first question traces its root back to Baird [1986]: if raising cash for bids were easy and there were enough competition among bidders, the liquidation procedure would be a valid alternative to reorganization. The model supports Baird [1986]'s claim. When the cash-auction procedure is efficient - modelled as a zero cleareance loss upon asset sales in liquidation - there is a healthy drop in the relative use of the Ch 11 procedure (-80%). Firms need to be more productive to reorganize and more firms get liquidated. This effect triggers a positive churning, that boosts median firms TFP (1.3%), output (1.1%) and, ultimately, consumption (0.6%). On the top of that, I show that shutting down Ch 11 when

Ch 7 is efficient is less costly.

The last exercise addresses a long-lasting question in the corporate bankruptcy literature: what is the optimal level of creditors' rights? Quoting Aghion et al. [1994], a good bankruptcy law 'efficiently' balances two *goals*: 1) the one of maximising the bonding value of the debt obligation (*ex-ante* efficiency), by adequately punishing the defaulter; 2) the one of maximising the social value of the firm (*ex-post* efficiency), by minimising the number of inefficient liquidations. In the attempts of isolating an optimal balance[5] to this trade-off, the theoretical literature has either sacrificed a general equilibrium perspective - by restricting its attention to partial equilibrium environments[6] - or[7] failed to provide a quantitative advice to the legislator. This exercise complements the literature on both dimensions. From a general equilibrium point of view, an increase in creditors' rights - modelled as a decrease in the bargaining power of firms - has positive macroeconomic effects: a 10% decrease in the firms' bargaining power, increases output by 0.9%.

Generally speaking, this paper adds to the macroeconomic literature that studies the impact of financial frictions on firm dynamics[8], macroeconomic aggregates and total factor productivity[9]. Most of these papers appraise default as financial frictions and, accordingly, model it in a very reduced form way. Conversely, I take seriously the phenomenon of corporate default, and in the spirit of Chatterjee et al. [2007] I use the lens of the bankruptcy law to characterise the default options of the firms. By doing so, instead of *indirectly* pinning down the financial frictions in the economy by targeting some aggregate feature of the economy, I *directly* gauge them down using estimates of the creditors recovery rates on corporate bankruptcy default.

Together with Corbae and D'Erasmo [2015], this paper firstly investigate in a firm dynamic model á la Hopenhayn[10] the macroeconomic implications of changes in the U.S. corporate bankruptcy law - where an increase in the bargaining power of the creditors yields higher recovery values, upon reorganization. Corbae and D'Erasmo [2015] study the implication of a longly debated bankruptcy reform suggested by Aghion et al. [1994] and recently proposed by the American Bankruptcy Institute. As a result, they document - among other aspects - an increase in consumption (2.32%), output (1.99%) and measured TFP (1.03%) after the adoption, due to cheaper borrowing, and better allocation of re-

[5] According to the weight which is put on each one of the two goals the bankruptcy law design classifies as more pro-debtor (*softer*) or more pro-creditors (*tougher*).

[6] As instance, focusing on particular asymmetric information environment, capital structure of the firms, degree of under-waterness of the firm. See Knot and Vychodil [2005] for a review.

[7] To the best of my knowledge, Biais and Mariotti [2009] represents the only work that studies the general equilibrium implications of changes in the bankruptcy law design.

[8] Among others, Cooley and Quadrini [2001], Jermann and Quadrini [2008], Jermann and Quadrini [2012].

[9] Among others, Moll [2014], Midrigan and Xu [2010].

[10] This paper adds to the vast literature on firm dynamics pioneered by Hopenhayn [1992], in which the author extends the long run industry equilibrium theory by introducing a concept of stationary equilibrium, which allows him to investigate the phenomena of entry, exit and heterogeneity in the size and growth rates of firms.

sources in the economy.

In line with the findings of Midrigan and Xu [2010][11], this paper complements the previous literature on endogenous equilibrium default [12], by gauging the importance of the extensive margin of entry/exit channel.
The paper is organised as follows. Section 1.2 discusses the salient features of the U.S. bankruptcy procedures. Section 1.3 reviews the empirical evidence on U.S. corporate bankruptcies. In Section 1.4 I introduce the model and in Section 1.5 I define and characterise the equilibrium. Section 1.6, 1.7, and 1.8 perform the quantitative analysis and Section 1.9 concludes.

1.2 The U.S. Bankruptcy Code

Article I, Section 8, of the United States Constitution authorizes the Congress to enact *uniform federal* laws on the subject of Bankruptcies. Under this grant of authority, the Congress enacted the Bankruptcy Code in 1978, a system of rules aimed at regulating insolvent incorporated firms. Pursuant to the U.S. Bankruptcy Code, corporations can take debt relief under two court-supervised procedures: liquidation and reorganization. *Liquidation* is a cash auction on the assets of the insolvent firm. The assets of the firm are sold to reimburse the creditors in a precise order: secured creditors, unsecured creditors and, eventually, the debtors. This procedure corresponds to the Chapter 7 of the U.S. Bankruptcy Code. Instead, the *reorganization* procedure disciplines the negotiation process between the debtor and its creditors upon a debt haircut and restructuring plan. This procedure corresponds to the Chapter 11 of the U.S. Code.

1.3 Empirical Evidence

I discipline the theory using a series of facts that relates firms characteristics with filing decisions and creditors recovery values.

First, I isolate a parsimonious set of facts from the analysis of Bris et al. [2006][13]. *Ceteris paribus*,

[11] Midrigan and Xu [2010] show that large differences in GDP per capita across countries can be accounted for the presence of financial frictions which misallocate resources (*intensive margin* channel), and distort entry and technology adoption decisions (*extensive margin* channel). The main finding of their paper is that the second channel is the one that seems to empirically matters.

[12] For instance, Arellano et al. [2012] show

[13] The study covers all the *corporate* bankruptcies filed in the Federal Bankruptcy Courts of Arizona and Southern New York from 1995 to 2001, made available to the *Pacer* (Public Access to Court Electronic Records) service. After cleaning the datasets from cases routinely dismissed or transferred to other courts, bankruptcy of subsidiaries, pre-packs bankruptcy (lasting as little as 2 weeks) 286 firms survives: 225 reorganization cases (Chapter 11) and 61 liquidation cases (Chapter 7) Bris et al. [2006] p.1256-1257. According to the authors *this is the largest and most comprehensive sample of corporate bankruptcies assembled for an academic paper*, Bris et al. [2006] p.1255.

- **Fact 1.** Tiny firms tend[14] to file for liquidation (Ch 7), big firms tend to file for reorganisation (Ch 11).
- **Fact 2.** More leveraged firms tend to file for reorganisation (Ch 11).
- **Fact 3.** Creditors' losses are lower in reorganization (Ch 11) than in liquidation (Ch 7).
- **Fact 4.** All else equal, creditors' losses increase in leverage

Fact 1 and 2 organize the evidence which relates the choice of the bankruptcy procedures to the characteristics of the firms. First of all, the propensity to file for reorganisation increases with the firm's size: defaulting firms with assets lower than 100000$ tend to file for liquidation (Ch 7), while bigger firms ($100K-$1M), tend to file for reorganization (Ch 11). Secondly, firms leverage matters: upon default, highly levered firms file for reorganisation[15] (Fact 2). Fact 3 and 4 relates creditors losses[16] to the type of bankruptcy procedure and firms' characteristics, respectively: creditors *fare significantly better* in Ch 11 reorganisation rather than in Ch 7 liquidations[17] (Fact 3); creditors recover less from firms that are more underwater (Fact 4).

Second, using the UCLA LoPucki Bankruptcy Research Database, 1980-2012 I document the following fact. Other things equal,

- **Fact 5.** Ch 11 creditors' recovery values do not depend on the outstanding liability at filing date.

In what follows I device a model that *explains* these facts either analytically (**Fact 3**, **Fact 4**, **Fact 5**, Section 1.5.1) or quantitatively (**Fact 1**, **Fact 2**, Section 1.7).

1.4 The Model

The economy is populated by firms, credit intermediaries and a household.

There is a continuum of firms. Firms are run by risk neutral managers which maximize the expected discounted stream of dividends. They combine capital and labour using a decreasing returns-to-scale technology, and experience uninsurable persistent idiosyncratic productivity shock. Firms articulate in two types: incumbents and entrants.

[14] In a probit sense.

[15] Bris et al. [2006] p.1259.

[16] Creditor losses are measured as the total pre-bankruptcy claim (normalised to 1) minus the recovery rate. The recovery rates is computed as the percent of the initial claim that is distributed by the court to the corresponding creditor in the case closure.

[17] Bris et al. [2006] report a median loss of 21% on the original claim under Chapter 11 versus a median loss of almost everything under Chapter 7 (Bris et al. [2006] p.1288).

The incumbents are the producing firms in the economy. They differ for the fixed capital scale of production, the fixed productivity and their histories. Incumbents finance investment and dividends using internal and external funds: retained profits, one-period non-contingent loans and equity issuance. Incumbents can choose to exit by repaying the debt or default on it. In accordance with the U.S. Bankrupcty Code, firms can renege on their debt obligations by filing for liquidation (Ch 7) or reorganization (Ch 11). If a firm files for liquidation, it relinquishes all its assets and profits (net of a liquidation loss) to the creditor and exits from the market. If a firm files for reorganization, it (nash-)bargains with the creditors over a debt reduction. If they find an agreement, the firm pays back the debt net of a haircut. During the bankruptcy procedure - which for simplicity lasts only one period - the firm incurs in a fixed cost.

In each period there is a positive mass of potential firms entering the economy with an initial level of assets and starting production with a time-to-build lag. After drawing a capital-scale, a permanent productivity level and an idiosyncratic productivity shock, potential entrants decides whether to *actually* enter or not. Actual entrants finance the capital scale by using assets, equity issuance, or debt. The capital scale remains fixed over the firm's life.

Firms have access to a competitive financial sector. Each financial intermediary offers a menu of loan sizes and interest rates to firms wherein each loan makes zero expected profits.

In conclusion, there is a representative household that owns the firms. The household saves in the credit market and supplies inellastically labour to the firm. It consumes out of the wage income, returns on savings and the aggregate amount of dividends distributed by the firms.

1.4.1 The Production Technology

Firms use capital, $k \in \mathrm{K} = [k_{min}, k_{max}] \subset \mathbb{R}_+$, and labour $n \in \mathrm{N} = [n_{min}, n_{max}] \subset \mathbb{R}_+$ to produce an homogeneous consumption good $y \in \mathrm{Y} \subset \mathbb{R}_+$ using a decreasing returns-to-scale (DRS) production technology,

$$y(x, n; k, z) = (zx)^{(1-\alpha\eta)}(k^{1-\alpha}n^{\alpha})^{\eta}$$

where: η is the decreasing return to scale parameter[18]; α is the value-added share of labour; $z \in \mathrm{Z} = [z_{\min}, z_{\max}]$ is the fixed productivity of the firms (drawn upon entry) and $x \in \mathrm{X} = [x_{\min}, x_{\max}] \subset \mathbb{R}_+$[19] is an uninsurable idiosyncratic shock. The idiosyncratic productivity follows a stochastic process defined on the measurable space $(\mathrm{X}, \mathcal{B}(\mathrm{X}))$ with

[18] Following Lucas [1978], the parameter $(1-\eta)$ is sometimes referred to as the *span of control.*

[19] In case $x_{min} = 0$ then it is possible that $y = 0$ in some periods. This creates some problem for the existence of the invariant distribution. Following Stokey et al. [1990b] and in line with the literature, I assume $x_{min} > 0$.

transition function $Q(x, \mathrm{d}x')$, where $\mathcal{B}(\cdot)$ denotes the Borel algebra on X. As standard in the literature, I further assume that $Q(x, \mathrm{d}x')$ is continuous on (x, x'), is decreasing in a first order stochastic dominance sense on x[20], and satisfies the strong Feller property.

The normalization parameter $(\cdot)^{1-\alpha\eta}$ on the actual productivity level, zx, ensures that firm's profit function after wage compensation $\pi(x; k, z)$ is linear on zx,

$$\pi(x; k, z) \equiv \max_n (zx)^{(1-\alpha\eta)}(k^{1-\alpha}n^{\alpha})^{\eta} - wn = zx\Theta\left[, \alpha, \eta, w(\mu)\right] k^{\gamma} \tag{1.1}$$

where

$$\Theta\left[, \alpha, \eta, w(\mu)\right] = \left[(1-\alpha\eta)\left(\frac{\alpha\eta}{w(\mu)}\right)^{\frac{\alpha\eta}{1-\alpha\eta}}\right] \qquad \gamma = \frac{(1-\alpha)\eta}{1-\alpha\eta}$$

and by FOC,

$$n^*(x; k, z) = zx\left(\frac{\alpha\eta k^{(1-\alpha)\eta}}{w(\mu)}\right)^{\frac{1}{1-\alpha\eta}} \tag{1.2}$$

As a result, at the optimal level of labour n^* output equals:

$$y(x; k, z) = zx\left(\frac{\alpha\eta}{w(\mu)}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} k^{\gamma} \tag{1.3}$$

The capital scale stays constant over life. Since capital depreciates at rate δ, investment equals $i = \delta k$[21]. Notice that as a consequence of the DRS technology, the depreciation process is more expensive for large firms. As a consequence of these assumptions on the physical technology, big firms have higher productivity in equilibrium (as in Melitz [2003]).

1.4.2 The Financing Technology

Incumbents finance investment using retained profits, one-period non-contingent loans and equity issuance[22]. Entrants cannot resort to retained profits. Let

$$g(d) = \left[\mathbb{I}_{\{d\geq 0\}} + \iota_I \cdot \mathbb{I}_{\{d<0\}}\right] \cdot d$$

denote the flow of dividends/equity issuance $d \in \mathrm{D} = [\underline{d}, \bar{d}] \subset \mathbb{R}$[23] to/from the household, with $\mathbb{I}_{\{y\}}$ denoting an indicator function that takes value 1 when y is true. Firms can issue equity by setting $d < 0$ and incurring in an additional proportional cost ι_I. Following the

[20] This is a property satisfied by many processes, especially the first order autoregressive process. The higher is the idiosyncratic productivity today, the more likely that it will be higher tomorrow.

[21] The result follows from the law of motion of capital $k' = (1-\delta)k + i$ and the fact that $k' = k$.

[22] To maintain tractable the state space, I do not consider the outright hierarchical layers of ownership (bonds, debentures, preferred equity, common equity) but just a neat pattern of layered debt and equity.

[23] The interested reader can refer to p. 39 for the precise definition of $\underline{d}, \bar{d}$.

literature, equity issuance is expensive ($\iota_I > 1$). This generates financing decisions that are in line with the pecking order theory: firms prefer retained profits and debt to equity.

Indeed, the presence of a default option yields a substantial departure of the loan-market arrangement from the Arrow-Debreu world. In turn this departure formalizes in the device of *firm specific* one-period non-contingent loan contracts, $\left(b', x, k, z, q_{b'}(x, k, z)\right)$, where $q : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to \mathrm{Q}$ is the real (vector-)valued pricing function in the space of continuous and bounded functions $\mathscr{C}_b^{[0,1]}$, with $\mathrm{Q} = [0, q_{\max}]^{|\mathrm{B}|} \subseteq \mathbb{R}^{|\mathrm{B}|}$, $0 \leq q_{\max} \leq 1$. In particular, a firm with characteristics (x, k, z) is allowed to save ($b' < 0$) or borrow ($b' > 0$) at the price $q_{b'}(x, k, z)$ the amount $b' \in \mathrm{B} = \{b_{\min}, \ldots, b_{\max}\} \subset \mathbb{R}$ where B is a finite set with cardinality $|\mathrm{B}|$, and with $b_{\min} < 0$ and $b_{\max} > 0$. This specification highlights the dependence of the loan price on three firms' *key characteristics*: 1) the actual productivity zx, 2) the size of assets, k, and 3) the size of the loan, b'. In presence of persistent idiosyncratic productivity shocks, if the actual x is high, next period productivity is likely to be high[24] too. In this case, the probability of default decreases, raising the price of debt. For the same reasoning, a higher permanent productivity z reduces the cost of debt service. Similarly, firms with more capital have a larger collateral, which tempers creditors' losses upon default and therefore price levels. Finally, larger loans increase the probability of default and reduce the loan price (higher interest rates).

To conclude, entrants face the same loan contracts.

Hereafter I refer to (d, b') as ordinary decisions to distinguish them from the extra-ordinary decisions represented by exit and default policies. Firms can choose to exit $\phi_X = 1$. Upon exit they can choose whether to repay the debt $\phi_D = 0$ or file for default, $\phi_D = 1$.

Let $B_b(x; k, z)$, the cash-flow correspondence of an incumbent (b, x, k, z) that chooses not to default as

$$\left\{(d, b') \in \mathrm{D} \times \mathrm{B} : d - q_{b'}(x, k, z)b' + k \leq \pi(x; k, z) - \chi_o + (1 - \delta)k - b\right\} \tag{1.4}$$

The next section explores the set of restrictions imposed by law on $B_b(x; k, z)$ when firms choose to default $(\phi_X, \phi_D) = (1, 1)$.

1.4.3 The Bankruptcy Law

The bankruptcy law is a technology that formalizes the legal ways by which firms can repudiate on their outstanding debt. According to the U.S. Bankruptcy Code, firms can file either for liquidation or for reorganisation. Accordingly, I model the default decision as a pair of indicator functions: (ϕ_D, ϕ_R). ϕ_D captures the *willingness* to default ($\phi_D = 1$) or not ($\phi_D = 0$). Conversely, the second indicator function determines the *form* of default: $\phi_R = 1$ if a firm selects bankruptcy reorganization and $\phi_R = 0$ if it selects bankruptcy liquidation.

[24] If productivity shocks were i.i.d., the interest rate would not depend anymore on the current x.

In the model, a *bankruptcy procedure* is described as: 1) a set of stipulations $S^R \in \mathbb{R}^2$ that the law imposes on firms' feasible choices; 2) a legal agreement on the amount of debt to be repaid; 3) the implications it has for the existence of the firm as a going concern. Hereafter $S^R_{\phi_D,\phi_R} \in \mathbb{R}^2$ denotes the set of restrictions imposed by law on the feasible ordinary choices of the firm, conditional on the default decision (ϕ_D, ϕ_R).

Then, I can characterize the budgetary implications of the bankruptcy choice (ϕ_D, ϕ_R) as the intersection between the cash-flow budget constraint of an incumbent which does not want to default (1.4) and the restrictions $S^R_{\phi_D,\phi_R}$ imposed by the law on (d, b')[25],

$$B_{\phi_X,\phi_D,\phi_R}(b, x; k, z) = B_b(x; k, z) \cap S^R_{\phi_D,\phi_R} \subseteq \mathrm{D} \times \mathrm{B}$$

By definition, $B_{0,0,0}(b, x; k, z) \equiv B_b(x; k, z)$ in (1.4).

Bankruptcy Liquidation

When a firm files for bankruptcy liquidation, it does not produce, it is prohibited to conduct any ordinary and extraordinary activity[26], $S^R_{1,0} = \varnothing$. Hence,

$$B_{1,1,0}(b, x; k, z) = \varnothing \tag{1.5}$$

The creditors seize the collateral of the firm, which consists of both profits and undepreciated capital

$$R^7(k) = (1 - \psi)(1 - \delta)k \tag{1.6}$$

suffering a liquidation clearance loss $\psi \in (0, 1)$, which captures in reduced form the well-documented presence of frictions in the cash-auction procedure[27]. In conclusion, once the firm is liquidated, it exits the market.

[25] Provided that the loan-market arrangements accommodate the enriched environment, see Section 1.4.2.

[26] The judge appoints a trustee to work on the interests of the creditors.

[27] As instance, the *financing problem* and *the lack of competition problem* (See Aghion et al. [1994]). The first problem relates to the difficulties in raising big amount of fundings in a brief amount time. The second problem depends on the lack of competition on the bidding sides. See Shleifer and Vishny [2011] for an amplification of the *financing problem* in recessions due to the congestion of the secondary markets (fire-sales).

Bankruptcy Reorganization

Upon filing for bankruptcy reorganization, a firm cannot distribute dividends[28], cannot save[29] suffers legal and administrative costs, χ_b[30], and obtains a debt haircut. These restrictions formalize[31] in:

$$\begin{aligned} B_{1,1,1}(b,x;k,z) &= S^R_{1,1}(b,x;k,z) \\ &= \Big\{(d,b') \in \mathrm{D}_{-} \times \mathrm{B}_{+} : \delta k \leq \pi - (\chi_o + \chi_b) - \alpha^*_b(x,k,z)\cdot b + \underbrace{(-d)}_{\text{Eq.Iss.}} + \underbrace{q_{b'}(x,k,z)b'}_{\text{DIP Financing}}\Big\} \end{aligned} \tag{1.7}$$

Notice, firms can borrow during reorganization. The extension of credit to the reorganize company (regulated by §364) represents a key feature of the Ch 11 procedure, sometimes referred to as debtor-in-possession financing (DIP financing).

In reorganization, creditors receive

$$R^{11}(b,x;k,z) = \alpha^*_b(x,k,z)\cdot b \tag{1.8}$$

where $\alpha : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to \mathrm{A}$ with $\mathrm{A} = [0,1]^{|\mathrm{B}|} \subseteq \mathbb{R}^{|\mathrm{B}|}$, is the real (vector-)valued recovery rate function. In particular, $\alpha^*_b(x,k,z)$ is the recovery rate which is negotiated upon in the nash bargaining between the firm (b,x,k,z) and the creditors, as explained in details in Section 1.4.5.

1.4.4 The Decision Problems

The timing of the model unfold as follows: i) incumbents observes the realization of the productivity shock and ii) decide whether to continue or to exit; iii) In case they exit ($\phi_X = 1$) they can choose whether to repay the debt or filing for default ($\phi_D = 1$); iii.a) if they file for liquidation ($(\phi_X,\phi_D,\phi_R) = (1,1,0)$) they do not produce and exit the market; iii.b) if they file for reorganization ($(\phi_X,\phi_D,\phi_R) = (1,1,1)$), they produce, bargain over the debt haircut and take financing decisions *jointly* with the creditors; if they continue ($\phi_X = 0$), then iii.c) they produce and make dividend, investment, financing decisions; iii.d) if they exit, they reimburse the creditors with the depreciated capital and distribute the rest as a dividend.

[28] The purpose of Ch 11 is to give time for restructuring the business and focusing investment towards profitable products. Therefore, diversions of funds - e.g. in the form of distribution of dividends - are ruled out.

[29] These first two stipulations model the fact that - in order to protect the creditors claims - most bankruptcy laws do not allow firms to divert funds (by distributing dividends or save). In the U.S., the privilege of creditors' claims over the ones of the filing shareholders is invoked by the Absolute Priority Rule.

[30] Chapter 11 expenses have two components: debtors' expenses and unsecured creditors' committee expenses.

[31] $Y_{-} = Y \setminus \mathbb{R}_{++}$, and $Y_{++} = Y \setminus \mathbb{R}_{-}$.

The Incumbents

A firm is identified as a 4-*upla*, $(b, x; k, z) \in \mathrm{B} \times \mathrm{X} \times \mathrm{K} \times \mathrm{Z}$. Let the (vector-)valued function $v : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to \mathbb{R}^{|\mathrm{B}|}$ in the space of continuous (vector-)valued functions $\mathscr{V}$, denote the expected discounted flows of profits of a firm with characteristics $(b, x; k, z)$. Hence, let the (vector-)valued maximum operator $(Tv)(x; k, z) = \left\{(Tv)(b, x; k, z) : b \in |\mathbb{R}^{|\mathrm{B}|}\right\}$ describe the optimization problem of the firms whose expected discounted profits are evaluated according to $v \in \mathscr{V}$. Then, $\forall b \in \mathrm{B}$ and for all $v \in \mathscr{V}$ we have

$$(Tv)(b, x; k, z) = \max_{\phi_X} \left\{ \underbrace{V^C(b, x; k, z)}_{\text{Continuation}} , \max_{\phi_D} \left\{ \underbrace{V^X(b, x; k, z)}_{\text{Exit}} , \max_{\phi_R} \left\{ \underbrace{V^R(b, x; k, z)}_{\text{Reorganization}}, \underbrace{V^L(b, x; k, z)}_{\text{Liquidation}} \right\} \right\} \right\} \tag{1.9}$$

where

$$V^X(b, x; k, z) = (1 - \delta)k - b$$

denotes the value at exit,

$$V^C(b, x; k, z) = \max_{d, b' \in \mathrm{D} \times \mathrm{B}} g(d) + \beta \cdot \int_{\mathrm{X}} v_{b'}(x'; k, z)\, Q(x, \mathrm{d}x') \tag{1.10}$$

$$\text{s.t.} \quad d - q_{b'}(x, k, z) b' \leq \pi(x; k, z) - \chi_o - \delta k - b \tag{1.11}$$

denotes the value of continuation, and where I normalise the value of a liquidation

$$V^L(b, x; k, z) = 0.$$

The reorganization value $V^R(b, x; k, z)$ is determined in the nash bargaining procedure between debtors and creditors, discussed in the next section.

Theorem 1.4.1. *i) There exists a unique $v^* \in \mathscr{V}$ such that: $v^* = (Tv^*)$. $\mathscr{V}$ is the set of continuous and bounded functions which satisfies the monotonicity properties A2-A3 (Appendix 1.A.1, p. 33). ii) The optimal correspondence $B^*_{\phi_X, \phi_D, \phi_R}(b, x, k, z)$ is compact-valued and upper hemicontinuous.*

Proof. See Appendix 1.A. □

1.4.5 The Reorganization Problem

In bankruptcy reorganization, a firm bargains with the financial intermediaries over the due recovery rate on the defaulted loan.

Let $\theta \in (0, 1)$ denote the bargaining power of the firms, which is homogenous across firm types. Let $\alpha^*(x, k, z) = \{\alpha^*_b(x, k, z) \in [0, 1] : b \in \mathrm{B}\} \subset \mathbb{R}^{|\mathrm{B}|}$, with $\alpha^* : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to$

$\mathrm{A} \equiv [0,1]^{|\mathrm{B}|} \subset \mathbb{R}^{|\mathrm{B}|}$ be a real (vector-)valued correspondence,

$$\alpha_b^*(x,k,z) = \arg\max_{a\in[0,1]} \left\{ S^f(a;b,x,k,z)^\theta \, S^c(a;b,x,k,z)^{1-\theta} \right\} \tag{1.12}$$

$$\text{s.t.} \quad \underbrace{S^f(a;b,x,k,z)}_{\text{Surplus of the Firm}} \geq 0, \qquad \underbrace{S^c(a;b,x,k,z)}_{\text{Surplus of the Creditors}} \geq 0 \tag{1.13}$$

where $S^f(a;b,x,k,z) : \mathrm{A}\times\mathrm{X}\times\mathrm{K}\times\mathrm{Z} \to \mathbb{R}^{|\mathrm{B}|}$ denotes the surplus of a firm which defaults and files for reorganization $(\phi_X, \phi_D, \phi_R) = (1,1,1)$,

$$S^f(a;b,x,k,z) = \max_{(d,b')\in\mathrm{D}_-\times\mathrm{B}_+} g(d) + \beta \cdot \int_{\mathrm{X}} v_{b'}(x';k,z)Q(x,\mathrm{d}x')$$

$$\text{s.t.} \quad \underbrace{d}_{\text{Eq.Iss.}} - \underbrace{q_{b'}(x,k,z)b'}_{\text{D.I.P. Financing}} \leq \pi(x;k,z) - (\chi_o + \chi_b) - ab - \delta k \tag{1.14}$$

and $S^c(a;b,x,k,z) : \mathrm{A}\times\mathrm{X}\times\mathrm{K}\times\mathrm{Z} \to \mathbb{R}^{|\mathrm{B}|}$ denotes the surplus of the credit intermediaries, defined as the recovery value under reorganization $a \cdot b$ net of the recovery value in case of liquidation (1.6)

$$S^c(a;b,x,k,z) = \underbrace{a \cdot b - R^7(k)}_{\text{Best Interest of Creditors Test}} \tag{1.15}$$

This specification of the surplus of the credit intermediaries formalizes a crucial requirements[32] for the court to approve the reorganisation plan. Pursuant to §1129 of Chapter 11[33] impaired class of claims or interests 'will receive or retain under the plan on account of such claim or interest property of a value, as of the effective date of the plan, that is not less than the amount that such holder would so receive or retain if the debtor were liquidated under chapter 7 of this title on such date'[34].

It is important to notice that firms and creditors make jointly financing, bargaining and dividend decisions in the attempt of maximising the over-all surplus. The possibility of issuing debt under bankruptcy reorganisation - known as debtor in possession financing - has represented an important novelty of the 1978 Bankruptcy Reform.

Theorem 1.4.2. *There exists a unique* α^*, *with* $\alpha_b^*(x,k,z) \in \mathscr{C}_b^{[0,1]}(\mathrm{X}\times\mathrm{K}\times\mathrm{Z})\ \forall b \in \mathrm{B}$, *which solves* (1.12).

Proof. See Appendix 1.B. □

In conclusion, let the value function of a firm filing for reorganization $V^R : \mathrm{X}\times\mathrm{K}\times\mathrm{Z} \to$

[32] Sometimes referred to as the 'best interest of creditors' test.

[33] Bankruptcy Code, p. 441.

[34] Clearly, the fact that the outcome of financial contract reorganization does depend on the liquidation values of the underlying assets is consistent with the empirical evidence of Benmelech and Bergman [2008].

$\mathrm{R}^{|\mathrm{B}|}$ be

$$V^R(b, x; k, z) \equiv S^f\left(\alpha_b^*(x, k, z); b, x, k, z\right)$$

1.4.6 The Credit Intermediaries

In the economy there is a competitive financial sector. Each risk neutral credit intermediary offers a set of firm-specific contracts $\left(b', x, k, z, q_{b'}(x, k, z)\right) \in \Omega\left(b', x, k, z\right)$.
Let $q : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \rightarrow \mathrm{Q} = [0, q_{\max}]^{|\mathrm{B}|} \subset \mathbb{R}^{|\mathrm{B}|}$ be a real (vector-)valued function in the space $\mathscr{C}_b^{[0,q_{\max}]}(\mathrm{X} \times \mathrm{K} \times \mathrm{Z})$ of continuous functions bounded between $[0, q_{\max}]$, with $0 \leq q_{\max} < 1$. Then, we can define $L : \mathrm{Q} \equiv [0, 1]^{|\mathrm{B}|} \subset \mathbb{R}^{|\mathrm{B}|} \rightarrow \mathbb{R}^{|\mathrm{B}|}$ as the real (vector-)valued pricing correspondence $(Lq)(x, k, z) = \{(Lq)(b', x, k, z) : b' \in B\} \in \mathbb{R}^{|\mathrm{B}|}$,

$$(Lq)(b', x, k, z) =$$
$$\begin{cases} \dfrac{1}{1+r} & b' \leq 0 \\ \dfrac{1}{b'(1+r)}\mathbb{E}_{x'|x}\left[\underbrace{(1-(1-\phi_X)\cdot\phi_D)}_{\text{No Default}}\cdot b' + \underbrace{\phi_D}_{\text{Default}}\left(\underbrace{\phi_R}_{\text{Ch 11}}\cdot R^{11}(b, x; k, z) + \underbrace{(1-\phi_R)}_{\text{Ch 7}}R^7(k)\right)\right] & b' > 0 \end{cases} \tag{1.16}$$

with $\phi_i = \phi_i(b', x'; k, z)$ $i = D, R$. In words, firms earn the risk-free interest rate, r, on their savings ($b' \leq 0$). Conversely, loans' prices depend on the endogenous probability that the firm will meet its debt obligation and the recovery rates upon default. In turn the recovery rates depend on the bankruptcy procedure.

Entrants

There is a large number, M, of ex-ante identical potential entrants that decides whether to pay or not a fixed entry cost, χ_E.
By paying the entry cost, the potential entrants draw their idiosyncratic permanent productivity, z, permanent capital scale, k, and the physical productivity shock, x, from the probability measure space $(\mathrm{X} \times \mathrm{K} \times \mathrm{Z}, \mathcal{B}(\mathrm{X} \times \mathrm{K} \times \mathrm{Z}), G_{x,k,z})$ where $G_{x,k,z}$ denotes the joint probability measure[35]. Upon the realisation of their idiosyncratic (x, k, z), potential entrants decide whether to actually enter ($\phi_E = 1$) or not ($\phi_E = 0$). If they decide to enter, then they finance their capital scale, k, by accessing firm specific not contingent external debt contracts $(b', x, k, z) \in \Omega(x, k, z)$[36] and, ultimately, by issuing equity. The problem of

[35] In the benchmark model I assume that the permanent productivity and the capital scale are drawn independently. In the quantitative analysis I relax this assumption and consider the effect of different correlation between the two.

[36] Notice how the bankruptcy law affects firms entry decision by changing the feasible set of external financing opportunities.

a potential entrant, can therefore be described as

$$V^E = \max_{\phi_E} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \phi_E \left[\max_{b' \in B} g(d) + \beta \int_{\mathrm{X}} v_{b'}(x'; k, z) Q(x, \mathrm{d}x') \right] G_{x,k,z}(\mathrm{d}x, \mathrm{d}k, \mathrm{d}z)$$

$$s.t. \;\; d + k \leq q_{b'}(x, k, z) b' \tag{1.17}$$

$$g(d) = d\mathbb{I}_{\{d \geq 0\}} + (\iota_E \cdot d)\mathbb{I}_{\{d<0\}} \tag{1.18}$$

Then, by assuming free entry, in equilibrium firms will enter until

$$V^E(w) \geq \chi_E \tag{1.19}$$

with equality if in steady state $M > 0$. Henceforth I will refer to the previous equation as the free entry condition (FEC). Similarly to Hopenhayn [1992] firms face a fixed cost of entry, but differently from the seminal paper there is an additional cost of debt service, $q_{b'}(x, k, z)b'$, which is heterogeneous across firms over the (x, k, z) dimensions and depends on the bankruptcy law.

In conclusion, given the DRS production technology and the proportional investment cost, δk, firms drawing a high capital scale will be able to finance it only if accompanied by a draw of a high permanent productivity. In different words, in the model economy larger firms are going to be the most productive ones.

Invariant Distribution

For each (k, z)-type incumbent firm we can define a transition function $G^{I,k,z} : (\mathrm{B} \times \mathrm{X}) \times \left(2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right) \to [0, 1]$ from the state (b, x) to the state $Z = Z^{b'} \times Z^{x'}$ as

$$G^{I,k,z}((b, x), Z) = \left[1 - \phi_X^{k,z}(b, x)(1 - \phi_D^{k,z}(b, x)\phi_R^{k,z}(b, x))\right] \mathbb{I}^{k,z}_{b'(b,x) \in Z^{b'}} \int_{Z^{x'}} Q(x, \mathrm{d}x')$$

where $Z^{b'}, Z^{x'}$ are the projections of $Z \in \left(2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right)$. This transition function captures the probability that firms in (b, x) - that do not exit or choose to reorganize - migrate to Z.

Let $G^{E,k,z} : \mathrm{X} \times (\mathcal{B}(\mathrm{X})) \to [0, 1]$ be the transition function of a (k, z)-type entrant, defined as

$$G^{E,k,z}(x, Z) = \mathbb{I}^{k,z}_{b'(x) \in Z^{b'}} \int_{Z^{x'}} Q(x, \mathrm{d}x')$$

Let μ a probability measure in the space $\Lambda \left(\mathrm{B} \times \mathrm{X} \times \mathrm{K} \times \mathrm{Z}, 2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X} \times \mathrm{K} \times \mathrm{Z})\right)$ of probability measures. Then, I can define the operator $(\Psi\mu)$:

$$(\Psi\mu)(Z) = \sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} G^{I,k,z}((b,x),Z)\ \mu(b,\mathrm{d}x,\mathrm{d}k,\mathrm{d}z)$$
$$+ M \int_{K} \int_{Z} \int_{X} G^{E,k,z}(x,Z)\ G_{x,k,z}(\mathrm{d}x,\mathrm{d}k,\mathrm{d}z) \tag{1.20}$$

1.4.7 The Household

The economy is populated by a unit measure of infinitely-lived, identical households, with preferences over streams of consumption - represented by an istantaneous Bernoulli utility function $u(C)$ - which discount the future at the same rate of the firms, β.
In each period the household is endowed with one unit of time that supplies inelastically. It further decides how much to consume, C, and how much to lend to the financial intermediary B', by solving

$$V_H(B;\mu) = \max_{\{C,B'\}} u(C) + \beta V_H(B') \tag{1.21}$$
$$\text{s.t.} \quad C + q_{\max} B' = w(\mu) + D + B$$

where D is the aggregate dividend, and $q_{\max} = \dfrac{1}{1+r}$, where r is the risk free interest rate. Then in steady state:

$$\beta = q_{\max} = \frac{1}{1+r}$$

1.4.8 The Aggregates of the Economy

The producing firms in the economy are the incumbents that do not exit or that file for Ch11 reorganisation[37]. Hence, the aggregate net output is given by:

$$
\begin{aligned}
Y = &\sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} [1 - \phi_X (1 - \phi_D \phi_R)]\, y(x; k, z) \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) \\
&+ \sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \phi_X (1 - \phi_D) \cdot (1 - \delta) \cdot k \; \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) \\
&- \sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \underbrace{[1 - \phi_X (1 - \phi_D \phi_R)] \cdot \chi_o}_{\text{Maintainance cost of operation}} + \underbrace{\phi_X \phi_D \phi_R \cdot \chi_b}_{\text{Legal cost}} \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) \\
&- \sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} [1 - \phi_X (1 - \phi_D \phi_R)] [\iota_I - 1] \cdot \mathbb{I}_{\{d^* < 0\}} \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) \\
&- M \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \phi_E [\iota_E - 1] \cdot \mathbb{I}_{\{d^* < 0\}} G_{x,k,z}(\mathrm{d}x, \mathrm{d}k, \mathrm{d}z) - \underbrace{M \cdot \chi_E}_{\text{Entry cost}}
\end{aligned}
\tag{1.22}
$$

The aggregate investment is given by:

$$
I = \sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} [1 - \phi_X (1 - \phi_D \phi_R)] \;\; \delta k \; \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) + M \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \phi_E k G_{x,k,z}(\mathrm{d}x, \mathrm{d}k, \mathrm{d}z)
\tag{1.23}
$$

and by resource constraint, aggregate consumption is given by:

$$
C = Y - I \tag{1.24}
$$

In equilibrium, by monotonicity of utility function the constraint holds with equality. Not liquidating incumbents distribute an aggregate dividend/equity issuance:

$$
\begin{aligned}
D = &\sum_{\mathrm{B}} \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} [1 - \phi_X \phi_D (1 - \phi_R)]\, g\left(d^*(x; k, z)\right) \mu(b, \mathrm{d}x, \mathrm{d}k, \mathrm{d}z) \\
&+ M \int_{\mathrm{K}} \int_{\mathrm{Z}} \int_{\mathrm{X}} \phi_E g\left(d^*(x; k, z)\right) G_{x,k,z}(\mathrm{d}x, \mathrm{d}k, \mathrm{d}z)
\end{aligned}
\tag{1.25}
$$

Let $a_{b',x,k,z}$ denote the aggregate amount of firm specific contracts $\left(b', x, k, z, q_{b'}(x, k, z)\right)$

[37] Entrants and liquidating firms do not produce.

issued,

$$a_{b',x,k,z} = \underbrace{q_{b'}(x,k,z)b'}_{\text{Amount of Loan granted}} \left[\underbrace{\int_{K\times\dots} \mathbb{I}_{b',x,k,z}\mu(b,\mathrm{d}x,\mathrm{d}k,\mathrm{d}z)}_{\text{Measure Incumbents asking}} + \underbrace{M \int \phi_E \mathbb{I}_{b'(x,k,z)=b'} G_{x,k,z}(\mathrm{d}x,\mathrm{d}k,\mathrm{d}z)}_{\text{Measure Entrants asking}} \right]$$

Then, the total aggregate demand of loan demand in the economy is

$$B^d = \sum_{B} \int_K \int_Z \int_X a_{b',x,k,z}\mu(b,\mathrm{d}x,\mathrm{d}k,\mathrm{d}z) \tag{1.26}$$

And in conclusion the aggregate demand of labour equals

$$N^d = \sum_{B} \int_K \int_Z \int_X n^*(x;k,z)\,\mu(b,\mathrm{d}x,\mathrm{d}k,\mathrm{d}z) \tag{1.27}$$

where $n^*(\cdot)$ is defined in (1.2).

1.5 Equilibrium

Definition. A steady-state competitive equilibrium is a set of prices $\{w^*, q^*, \alpha^*\}$, a measure μ^*, a mass of potential entrants M^*, the incumbents policies $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, the entrants policy functions $\{\phi^{E,*}, b_e'^{,*}, d_e^*\}$, and the household decisions $(C^*, B'^{,*})$ such that:

1. given $(w^*, \alpha^*, q^*, v^*)$, then $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$ solve the incumbents problem (1.9)
2. given $(w^*, \alpha^*, q^*, v^*)$ and $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, then $\{\phi_E^*, b_e'^{,*}, d_e^*\}$ solve the entrants problem (1.17)
3. given $(w^*, \alpha^*, q^*, \mu^*)$, $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, and $B'^{,*}$, then C^* solves the household problem (1.21)
4. given (w^*, q^*, v^*, μ^*), and $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, then α^* is the nash bargain solution (1.12)
5. given (w^*, q^*, α^*), $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, then q^* satisfies the zero-profit condition (1.16)
6. given $(w^*, q^*, \alpha^*, v^*)$, $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, then w^* satisfies FEC (1.19)

7. given $(w^*, q^*, \alpha^*, v^*)$, $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, then $\mu^* = \Psi_{(w^*,\alpha^*,v^*,q^*)}M$, $\forall M$

8. given $(w^*, q^*, \alpha^*, \mu^*)$, $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, and C^*

   8.1. M^* is such that labour market clears, $N^s = N^d(M^*)$, where N^d defined in (1.27)

   8.2. $B'^{,*}$ is such that the loan market clears, $B^d = B'^{,*}$, where B^d defined in (1.26)

1.5.1 Characterization of the Equilibrium

Theorem 1.5.1. *Let* $\alpha^* \in \mathscr{C}_b^{[0,1]}(\mathrm{X} \times \mathrm{K} \times \mathrm{Z})$ *be the solution to the Nash Bargaining problem* (1.12)*. Then for any* $(b, x, k, z) \in \mathrm{B} \times \mathrm{X} \times \mathrm{K} \times \mathrm{Z}$*, if the nash bargaining surplus*

$$S(x,k,z) \equiv \pi(x;k,z) - \delta k - (\chi_o + \chi_b) + q_{b^{*\prime}}(x,k,z)b^{*\prime} + \frac{\beta}{\iota} \cdot \int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x') - R^7(k) \geq 0$$

then an interior solution $\alpha_b^*(x,k,z)$ *exists and equals:*

$$\alpha_b^*(x,k,z) = \frac{R^7(k)}{b} + (1-\theta) \cdot \frac{S(x,k,z)}{b} \tag{1.28}$$

Proof. See Appendix 1.C. □

The following corollary follows from inspection of (1.28).

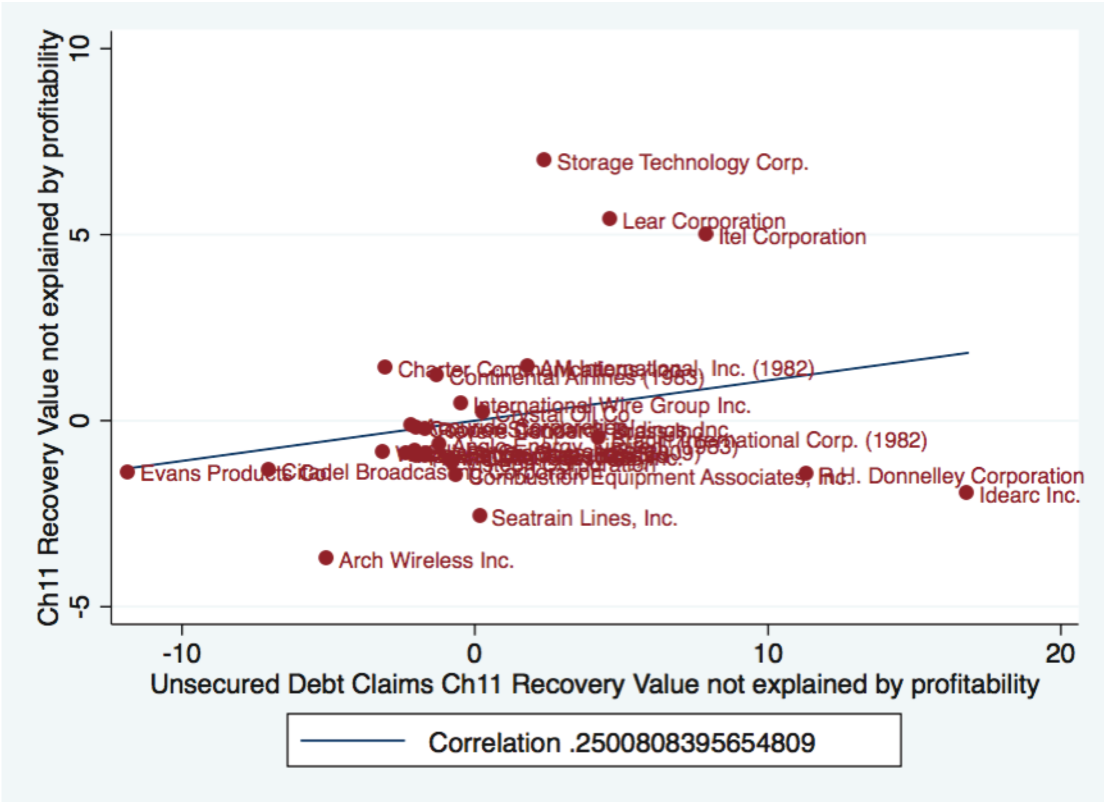
Corollary 1.5.2. *Given k, Ch 11 recovery values* $\alpha \cdot b$ *are greater (equal) to Ch 7 recovery values,* $R^7(k)$ *(**Fact 3**).*

Proof. In an interior solution $S(x,k,z) \geq 0$. Then the result follows by inspection of (1.28). □

Fact 3 posits that, ceteris paribus, creditors' losses in reorganization are lower than in liquidation. Corollary 1.5.2 rationalizes this result as a by-product of the mathematical formalisation (equation (1.15)) of §1129 of Chapter 11[38], sometimes referred to as the 'best interest of creditors' test. This section requires - as a *necessary condition* for the judge to approve the reorganisation plan - that the amount received by the impaired classes of creditors under Chapter 11 reorganisation to be no less than the amount they would receive if the debtor were liquidated under Chapter 7.

[38] Bankruptcy Code, p. 441.

Figure 1.1: Ch 11 Recovery Value and Debt (Unsecured): Orthogonal Components



Note: The vertical and horizontal axis reports, respectively, the components of chapter 11 recovery value $\alpha \cdot b$ (real total distributions to unsecured non-priority creditors, at confirmation prices) and outstanding debt b (real total unsecured, non-priority claims, at filing prices) that are not explained by: Average Net Income (average of real net income for last year ending before case filing and after case disposition), Days in Reroganization, Share of Claims to Secured Creditors over total Assets Petition at filing. Corr. reports the correlation among the residuals. *Source:* UCLA LoPucki Bankruptcy Research Database, 1980-2012.

Corollary 1.5.3 formalizes another important testable implication of Theorem 1.5.1.

Corollary 1.5.3. *$\alpha_b^*(x,k,z)b$ is independent of b: the amount to repay does not depend on the total level of debt, or leverage, but only on (x,k,z) (**Fact 5**).*

Proof. See Appendix 1.D. □

Once in reorganization, recovery values depend on the ability to generate future surpluses and do not depend on the outstanding debt. This result follows from the forward looking nature of the bargaining process.

To test this prediction (**Fact 5**), I collect data from the UCLA LoPucki Bankruptcy Research Database, 1980-2012 and run the OLS cross-sectional regression

$$(\alpha \cdot b)_i = \beta_0 \; + \underset{(.09)}{\underset{.108}{\beta_b}} \cdot b_i \; + \underset{(.181)}{\underset{.487}{\beta_I}} \cdot \text{Average Net Income}_i + \; \beta_X \cdot X_i \; + \; \epsilon$$

where $\alpha \cdot b$ is the real total distributions to unsecured non-priority creditors, b is the real total unsecured, non-priority claims, Average Net Income is the average of real net income for last year ending before case filing and after case disposition, and

$$X = \{\text{Days in Reroganization, Share of Claims to Secured Creditors}\},$$

is a set of regressors that capture the complexity of the reorganization procedure.

Despite Ch 11 recovery values are highly correlated with the unsecured debtors claim (0.72), the correlation disappears once we control for firms' income (and variables that proxies for reorganization complexity). Figure 1.1 uses the Frisch-Waugh-Lovell theorem to highlights the weak (and statistical insignificant) partial effect of unsecured debtors claim on the Ch 11 recovery value $\alpha \cdot b$.

Corollary 1.5.4. *Ch 7 and Ch 11 recovery rates decrease with the amount of outstanding debt (**Fact 4**).*

Proof. By definition, Ch 7 recovery values $R^7(k)$ (equation (1.6)) are independent on the level of debt. By corollary 1.5.3, Ch 11 recovery values $\alpha^* \cdot b$ are independent on the level of debt (in an interior solution for $\alpha^* \cdot b$). The result follows, by dividing these quantities by b. □

While the fact that Ch 7 recovery rates are decreasing in debt does not come as a surprise, the fact that Ch 11 recovery rates decrease in debt results from the forward looking nature of the reorganisation process.

Ultimately, Corollary (1.5.3) unveils an important implication of the forward looking nature of the reorganisation process. Firms with different outstanding debt, b, but same productivity and assets (x, k, z) repay the same amount $\alpha^* b$. It follows that their value is independent on the outstanding debt, as captured by the following corollary.

Corollary 1.5.5. *If there exists an interior solution for $\alpha^* \cdot b$, the value of the firm in reorganisation $V^R(b, x; k, z)$ (eq. (1.14)) is independent of b.*

Proof. By inspection of (1.14), the value function of a firm who seeks to reorganise $V^R(b, x; k, z)$ depends on b through $\alpha_b^*(x, k, z) \cdot b$. Since by Corollary 1.5.3 $\alpha_b^*(x, k, z) \cdot b$ is independent on b (in an interior solution) the result follows. □

Fact 1 and 2 are equilibrium results, which are quantitatively accounted for in Section 1.7.

1.6 Calibration

The economy is calibrated over the period 1979-2012. One period in the model corresponds to one year. The model has 23, parameters: physical technology (α, η, δ, χ_o), financing technology, ι_I, bankruptcy technology (ψ, θ, χ_b), entrants (χ_E, ι_E), discounting (β, r), labour supply, N, and uncertainty $\{(\bar{\mu}_{\ln x}, \rho_{\ln x}, \sigma_\epsilon), (\mu_{G(x)}, \sigma_{G(x)}), (\mu_{G(z)}, \sigma_{G(z)}), (\kappa_k, k_{\min}), p_X\}$, whose parameters are discussed, in details, in the next section.

I use estimates or impose restrictions on 12 parameters and structurally estimate the rest.

1.6.1 Uncertainty

To calibrate the model, let us impose more structure on the uncertainty governing the model economy.

The log-idiosyncratic productivity shock $\ln x_t$ follows an AR(1) process

$$\ln x_{t+1} = \rho_x \ln x_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N\left(0, \sigma_\epsilon^2\right) \tag{1.29}$$

I approximate the process by a discrete-state Markov chain (9 points), using Gauss-Hermite nodes and weights and applying the Tauchen and Hussey [1991] weights correction in order to account for the persistency.

For what concerns the uncertainty at entry, $G_{x,k,z}$, I make the following assumptions: the permanent and persistent idiosyncratic productivity shocks are drawn from log-normal distributions, $G_z(\mu_{G(z)}, \sigma_{G(z)})$ and $G_x(\mu_{G(x)}, \sigma_{G(x)})$; the fixed capital scale is drawn from a pareto distribution $G_k((\kappa_k, k_{\min})$. By assuming independence across these dimensions, I have $G_{x,k,z} = G_r \cdot G_x \cdot G_k \cdot G_z$.

1.6.2 Restricted Parameters

I start with the **physical technology**. Following Gilchrist et al. [2013], I set the value-added share of labour in the production function $\alpha = 0.7$, and the estimated decreasing return to scale parameter $\eta = 0.85$[39]. The real risk-free rate and the annual risk-manager discount factor are set to $r = 0.04$ and $\beta = \dfrac{1}{1+r} = 0.9615$, as standard in the literature. Hence, I normalize the labour supply $N^s = 1$. For what concerns the **financing technology**, I assume that entrants and incumbents face the same equity issuance cost, $\iota_I = \iota_E$.
I then set ψ to capture the percentage of *post*-bankruptcy asset value which is lost in the cash auction procedure due to legal[40] expenses or fire saling. This operation involves two complications. The first one regards the estimate of *post*-bankruptcy assets, which are defined as the percent recovery rates by creditors times amount owed to creditors, plus total legal fees disclosed and reimbursed by the court.

[39]These parameters are consistent with the literature (e.g. Barseghyan and DiCecio [2011]). In turn, this parameters specification imply a decreasing returns to scale parameter over physical capital $\gamma = 0.63$ consistent with the lower bound of reasonable parameters for the class of Cobb-Douglas production function (e.g. Arellano et al. [2012]).

[40]Namely, trustee expenses, accountant expenses, debtor attorney expenses.

Table 1.1: Parameters of the Model

		Physical Technology	
α	0.70	Value-added share of labour	Gilchrist et al. [2013]
η	0.85	Production Function Returns to Scale	Gilchrist et al. [2013]
		Financing Technology	
ι_E	ι_I	Equity issuance cost entrants	Restrictions
		Bankruptcy Technology	
ψ	0.83	Liquidation Legal Cost	Bris et al. [2006]
		Economy	
β	0.96	Subjective Discount Factor	FOC
r	0.04	Real Risk-Free Interest Rate	FRED
N^s	1.00	Labour Supply	Normalization
		Uncertainty	
$\bar{\mu}_{\ln x}$	0	Unconditional mean of $\ln x_t$	Standard
$\mu_{G(x)}$	0	Expected persistent productivity $\ln x_t$	Restriction
$\sigma_{G(x)}$	$\sigma_\epsilon / \sqrt{1-\rho^2_{\ln x}}$	Standard deviation of persistent productivity $\ln x_t$	Restriction
$\mu_{G(z)}$	0	Expected permanent productivity $\ln z_t$	Restriction
$\sigma_{G(z)}$	$\sigma_\epsilon / \sqrt{1-\rho^2_{\ln x}}$	Standard deviation of permanent productivity $\ln z_t$	Restriction

The reason is that the final Chapter 7 bankruptcy declaration omits direct seizures of assets by secured creditors, and therefore impairs the estimation of Chapter 7 creditor recovery rates. Bris et al. [2006] circumvent this problem providing a pessimistic and optimistic statistics. Second, in the model there is no distinction between *pre*-bankruptcy assets (i.e. assets declared in the initial bankruptcy petition filing as 'value of assets') and *post*-bankruptcy assets, since the Chapter 7 procedure takes a single period. To control for these issues, using a brute force approach, I obtain $\psi = 0.83$ as the arithmetic average between the median expense to post-bankruptcy asset ratio under optimistic (9.6%) and pessimistic (100%) estimates[41] weighted for the median post-pre bankruptcy assets ratio $\left.\frac{A_{post}}{A_{pre}}\right|_{\mathrm{X}}$ under optimistic (38%) and pessimistic (0.8%) estimates[42] as shown in the formula

$$(1-\psi) = \frac{\left.\frac{A_{post}}{A_{pre}}\right|_{\mathrm{Pess}}(1-\psi_{\mathrm{Pess}}) + \left.\frac{A_{post}}{A_{pre}}\right|_{\mathrm{Opt}}(1-\psi_{\mathrm{Opt}})}{2} = \frac{0.008(1-1)+0.38(1-0.096)}{2}$$

For what concerns the **uncertainty at entry**, the initial idiosyncratic productivity shocks x' are drawn from the long-run log normal distribution, $G_x(0, \sigma_\epsilon/\sqrt{1-\rho_x^2})$. Hence, I assume $G_z = G_x$, and I discretize the permanent idiosyncratic productivity into 3 levels associated to the conditional expectation of z falling in one of the following intervals: $[0, x_{20th}]$, $[x_{20th}, x_{80th}]$, $[x_{80th}, \infty]$, where x_{qth}, denotes the qth percentiles. By so doing I tie the cross-sectional distribution properties of the permanent efficiency with the long-run property of the efficiency process estimated in the data.

Table 1.1 reports the parameters restrictions.

1.6.3 Estimated Parameters

I estimate 11 parameters by minimizing the weighted sum of squared residual between a set of moments computed in the model, $m(\underline{\theta})$, and in the data, $\hat{m}$. I choose 34 moments that are a priori informative[43] about the firms distribution and the phenomenon of corporate bankruptcy default. Table 1.2 reports the results of the estimation.

1.7 Quantitative Results

In order to test the calibration accuracy, I use the model to replicate the analysis performed by Bris et al. [2006]. I simulate an economy populated by 45397 firms with the same characteristics of the long run invariant distribution. The number of firms is chosen such that in steady state I have 286 firms defaulting (0.63% default rate), as in the sample

[41] Bris et al. [2006], p. 1280.

[42] Bris et al. [2006], p. 1265.

[43] Heuristically speaking the moments are informative about the unknown parameter if they are sensitive to its changes.

Table 1.2: Simulated Method of Moments Estimation

Target	**Data**	**Model**	**Parameter**		**Description**
E_i[Exit Rate]	0.0712	0.0312	p_X	0.02	Exogenous Prob Exit
E_i[Default Rate]	0.63 %	0.13 %	χ_o	198046.88	Maintenance Cost
$E_i[C_L/B]$	0.16	0.9901	χ_b	129296.88	Legal Costs
$q_{i,50}[\alpha^*]$	0.5309	0.6155	θ	0.92	Firms Bargaining Power
$E_i[Y/A]$	0.32	2.3584	$k_{\min}$	8632.81	Lower Bound k
$\sigma_i[B/A\vert$ Incumbents]	0.2030	0.2751	κ_k	0.58	Pareto Exponent G_k
$E_i[B/A\,\vert$ Entry]	0.2064	0.9513	ι_I	0.60	Equity Issuance Cost Entrants
$E_i[V/A\,\vert$ Entry]	4.7208	5.1426	χ_E	365234.37	Entry Cost
$E_i[B/A\vert$ Incumbents]	0.1873	0.8661	δ	0.02	Depreciation Rate
$\sigma_i[B/A\,\vert$ Entry]	0.2435	0.3853	σ_ϵ	0.17	Volatility of innovation of $\ln(x)$
$q_{i,50}[B/A\vert$ Incumbents]	0.1247	0.7972	$\rho_{\ln x}$	0.81	Persistency of $\ln(x)$ AR(1)

Note: The first and second column report the structural parameters of the model and their description. The third column reports the targeted statistics: $E[\cdot]$ denotes time series averages, while $E_i[\cdot]$, $\sigma_i[\cdot]$ and $q_{x,i}[\cdot]$ denote the time series averages of, respectively, cross-sectional averages, standard deviations and cross-sectional x-percentiles. The Data column reports the moment computed in the data (firms ratios are trimmed at 1 and 99 percentiles). *Source:* Compustat North-America Fundamentals Annual, 1979-2012. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

documented by Bris et al. [2006].

Hence, as in Bris et al. [2006], I run a probit on the choice of the chapter (dummy equals 1 in case firms choose to file for Chapter 11) over the size of the firms K and leverage B/K, and an OLS regression on the creditor total recovery rates over K, B/K and the chapter choice, Ch 11.

Table 1.3 reports the results. The model is consistent with the fact that bigger and more leveraged firms tend to file (in probit sense) to chapter 11 (**Fact 1** and **Fact 2**). Moreover, total recovery rates are higher under Chapter 11 than Chapter 7 (**Fact 3**) and decreasing in leverage (**Fact 4**). Differently from the data, recovery rates increase in the firm's size.

1.8 Quantitative Analysis

In this section I perform four exercises. First, I use the model to assess the macroeconomic implications of the introduction of the reorganization procedure (Section 1.8.1). Second, I deepen the analysis and look at the economic relevance of a particular feature of the reorganization process: the debt in possession financing (Section 1.8.2). I then conclude by addressing two of the main organizing questions of the last two decades of corporate bankruptcy literature: What is the optimal level of creditor rights (Section 1.8.4)? Do we need Ch 11 in a world where Ch 7 is efficient (Section 1.8.3)?

Table 1.3: Determinants of Chapter Choice and Creditors' Losses

	Choice of Chapter	*Creditors Recovery*
	Ch 11	Tot. Recovery Rate
K	7.27	0.105
	(0.22)	(0.002)
B/K	0.28	-0.027
	(0.04)	(0.001)
Ch11	-	0.341
		(0.002)

1.8.1 The Economic Value of Ch 11

The reorganization process represents the main novelty of the 1978 Corporate Bankruptcy Code. In this section I use the model to study what would have happened if Ch 11 had never been introduced. To do so, Table 1.4 compares the calibrated economy, with an economy without Ch 11. First of all the economy records an important substitution of the bankruptcy procedure: Ch 7 default rates more than triple.

The cost of debt increases by 5 basis points. Firms demand less labour, causing a contraction in the number of incumbent firms −1.2%. Because of selection, firms at entry have to be more productive: median TFP increases by 1.44%. Net output falls by 1%, similarly consumption 0.7%.

Table 1.4: Steady state comparison with an Economy without Ch 11

Steady state comparison with an Economy without Ch 11	
	%
Bankruptcy Composition	
Fraction of Ch 7 defaults	250
Debt Schedules	
Average Debt Prices	-0.06
Firms Distribution	
Measure of Incumbents	-1.19
Median Employee (per firm)	-2.6
National Income Accounting	
TFP	1.44
Output (Y)	-1.07
Consumption (C=NY-I)	-0.72

1.8.2 The Debtor in Possession Financing

The debtor-in-possession financing (i.e. the right to borrow during reorganization, §364) was firstly introduced with the 1978 US Corporate Bankruptcy Code. Since then, the practice has experienced an unprecedented growth, fostered by a vested interest of financial institutions on high-yields investment opportunities and the development of the junk bond market[44]. Up to the financial crises, when the high-yield bond markets froze (Martin et al. [2009]).

This section investigates the general equilibrium implications of shutting down the debtor-in-possession financing channel. Table 1.5 compares the calibrated economy with an economy where firms cannot borrow in reorganization. The reorganization procedure becomes less attractive than the liquidation alternative: the fraction of Ch 11 defaulters drops by 70%, while the one of Ch 7 defaulters increase by 25%. As a result, default becomes more costly: median Ch 11 recovery values drop by 34%, and the equilibrium average interest rates increase by 6 basis points. Firms demand less labour, at any quintile of the distribution (median -1.2%), causing a reduction in the measure of incumbent firms (-0.4%). Overall, the steady state level of output and consumption drop significantly

[44]"The discovery that vast profits could be made by buying distressed debt began to resonate with Wall Street. You could buy claims at a steep discount from frustrated creditors and, hopefully, within a relatively short period, realize double digit returns" (cit, Miller [2007]).

($\sim$ 0.5%), accompanied by a drop in average total factor productivity (0.13%), due to the churning of large productive firms.

Table 1.5: Steady state comparison with an Economy without DIP financing

Steady state comparison with an Economy without DIP financing	
	%
Bankruptcy Composition	
Fraction of Ch 11 defaults	-68
Fraction of Ch 7 defaults	25
Debt Schedules	
Median Ch 11 Recovery Rates	-34
Average Debt Prices	-0.06
Firms Distribution	
Measure of Incumbents	-0.39
Median Employee (per firm)	-1.20
National Income Accounting	
Output (Y)	-0.49
Consumption (C=NY-I)	-0.44

1.8.3 The Economic Value of Ch 7

If raising cash for bids was easy and there was enough competition among bidders, the liquidation procedure would be a valid alternative to the reorganization ones (Baird [1986], Aghion et al. [1994]). Unfortunately, these conditions are often not met.

To study the general equilibrium implications of changes in the efficiency of the liquidation procedure, Figure 1.3 plots aggregate output in function of the clearance loss in cash auction procedures, ψ. When $\psi = 0$ output [consumption] is 1.1% [0.64%] higher, suggesting economically relevant gains from reducing frictions in the cash auction procedure. Table 1.6 explores the firms' dynamics forces behind this result. First of all, there is a significant drop in the relative use of the Ch 11 procedure (-80%). Less reorganization meets the requirements imposed by the best interest of creditors test (by §1129, creditors should recover in Ch 11 at least as much as under Ch 7), and more firms get liquidated. Firms need to be more productive to reorganize. This result in a positive churning effects, that boosts median firms TFP (1.27%)

Table 1.6: Steady state comparison with an Economy with $\psi = 0$

Steady state comparison with an Economy with $\psi = 0$		
	Ch 11 %	No Ch 11 %
Bankruptcy Composition		
Fraction of Ch 11 defaults	-81.20	-100
Debt Schedules		
Median Ch 11 Recovery Rates	35.10	-
Firms Distribution		
Measure of Incumbents	1.72	-0.98
Median Employee (per firm)	-2.60	0.94
National Income Accounting		
TFP	1.27	0.70
Output (Y)	1.07	-0.06
Consumption (C=NY-I)	0.64	-0.20

In addition, the model supports Baird [1986]' claim that in a world where the liquidation process is efficient Ch 11 loses economic relevance. To see this, column two of Table 1.6 reports the effect of shutting down Ch 11 in an economy where $\psi = 0$. The loss in terms of output and consumption are negligible [and gains in terms of TFP] are negligible with respect to the ones displayed in the baseline economy ($\psi = 0.83$, Table 1.4), suggesting that in an economy with small frictions in the cash auction procedure Ch 11 loses part of its role. Notice that the last estimate is a conservative measure of the value of Ch 7. Bidders in the model cannot bid for the going-concern value of the firm, and the recovery value under Ch 7 arise from selling the assets (that are not re-use in a new firm).

To sum up, we draw the conclusion that improvements in the efficiency of the liquidation procedure have economically relevant general equilibrium consequences that dwarf the role of the reorganization procedure.

1.8.4 The Creditor Rights Protection

In the years following its enactment, the 1978 Bankruptcy Code was subject to a harsh criticism from the credit institutions for its pronounced *pro*-debtors character.

In this section I investigate the quantitative implications of changes in the bargaining power of the firm on the economy to shed light on this debate. An increase in the bargaining power of the firm lowers the expected creditors recovery, making more expensive for the firms to borrow. All the rest equal, the reduction in the cost of the debt service has a twofold effect. On one side it makes more difficult for inefficient firms to stay, producing upward pressure on aggregate TFP. On the other side, it produces an endogenous credit rationing: aggregate debt drops and as a consequence there is shrink in the amount of firms in the economy. Meanwhile, the reduction in profitability reduces the measure of firms who is willing to enter. All together these dynamics produce a decrease in the wage, which in turn puts downward pressure on aggregate TFP. To attach some figure to this reasoning, Figure 1.2 plots the aggregate output in the economy for different values of the bargaining power θ. A 10% decrease in the firms' bargaining power, increases output by 0.9%. Notice that when the firms' bargaining power is low enough, output becomes less sensitive to changes in the creditors rights protection.

1.9 Conclusion

In this paper I build and characterize a general equilibrium firm dynamics model in which the default option replicates salient features of the U.S. Bankruptcy Code: distressed firms can voluntarily file either for liquidation (Chapter 7) or reorganization (Chapter 11).

In the theoretical analysis, I show that the model rationalizes the fact that recovery values under Ch 11 are independent on the initial level of debt (Fact 5), the fact that creditors' losses are lower in reorganization than in liquidation (Fact 3), and the fact that, all else equal, creditors' recovery rates decrease in leverage (Fact 4).

I merge firm's level accounting data from Compustat North-America Fundamentals Annualand bankruptcy information from the UCLA LoPucki Bankruptcy Research Database, database, and calibrate the model to the U.S. economy from 1979-2012. The model is consistent with the fact that small firms tend to file for liquidation, while big firms tend to file for reorganisation (Fact 1), and the fact that - ceteris paribus - more leveraged firms tend to file for reorganisation (Fact 2).

In the quantitative analysis, I use the model to perform four policy counterfactual experiments.

In the first exercise, I investigate what would have happened if Ch 11 had never been introduced in the 1978 reform of the Corporate Bankruptcy Code (Section 1.8.1). Results are striking: median TFP increase by 1.44%, net output falls by 1% (similarly consumption, by 0.7%).

In the second exercise, I quantify the relevance of a particular feature of the Ch 11 reorganization procedure: the debtor in possession financing - the right to borrow during reorganization, (Section 1.8.2). The practice of debtor-in possession (DIP) financing boomed with the development of the junk bond market during the 1990s (Miller [2007]). Nonetheless, with the onset of the financial crises the high-yield bond market froze, dras-

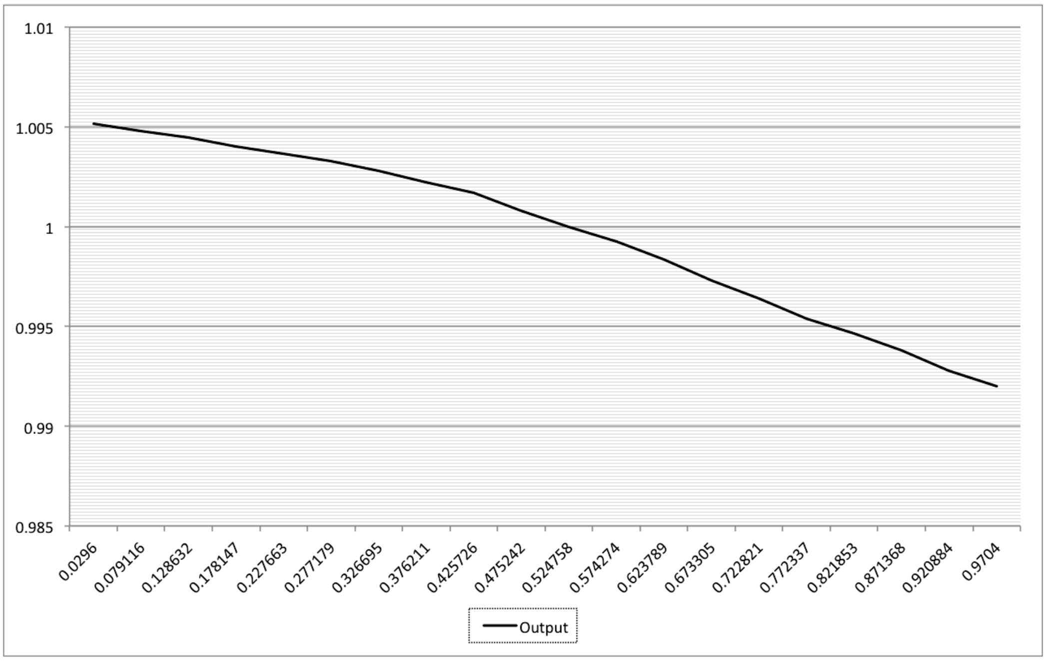


Figure 1.2: Aggregate output over cash auction clearence loss, ψ

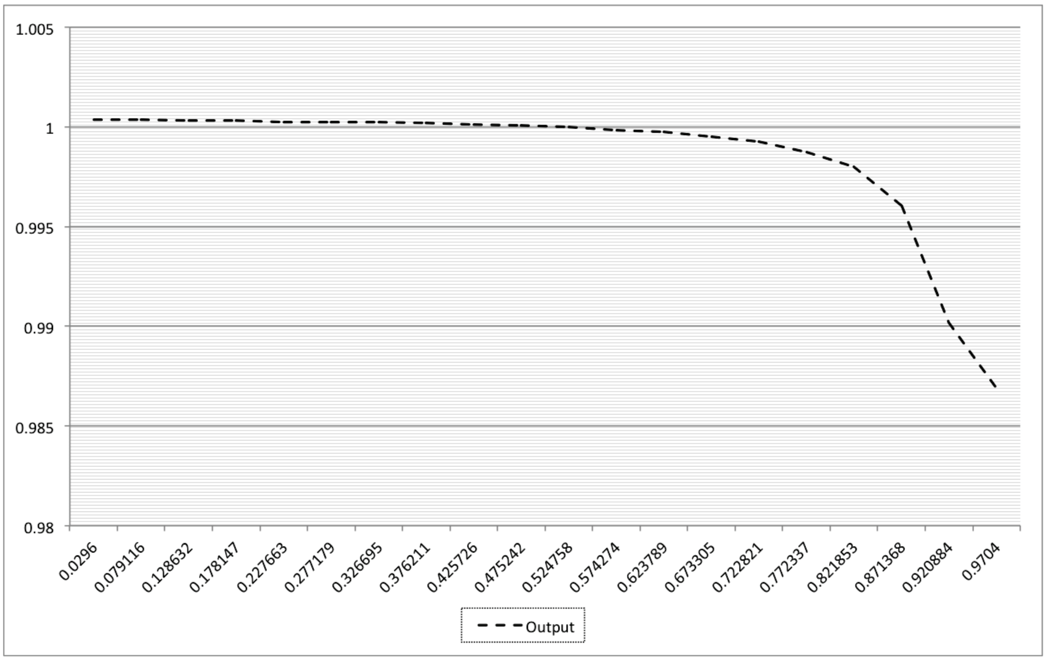


Figure 1.3: Aggregate output over bargaining power of firm, θ

tically reducing DIP fundings (Martin et al. [2009]). To study the general equilibrium implication of the junk-bond market freezing, I simulate an economy where firms cannot borrow during reorganization. The model suggests a drop in the relative use of the Ch 11 procedure (Ch 11 defaulters drop by 70%. Default becomes more costly: median Ch 11 recovery values drop by 34%, and the equilibrium average interest rates increase by 6 basis points. Despite TFP does not vary much (drop of 0.13%), output and consumption fall significantly ($\sim 0.5\%$) due to the churning of large productive firms.

In the last two exercises I address the following questions: Do we need Ch 11 in a world where Ch 7 is efficient (Section 1.8.3)? What is the optimal level of creditor's rights (Section 1.8.4)? The first question, traces its root back to Baird [1986]: if raising cash for bids was easy and there was enough competition among bidders, the liquidation procedure would be a valid alternative to the reorganization Aghion et al. [1994]). The model supports Baird [1986] claim. In world where the cash-auction procedure is efficient, there is a healthy drop in the relative use of the Ch 11 procedure (-80%). Firms need to be more productive to reorganize and more firms get liquidated. This effect triggers a positive churning, that boosts median firms TFP (1.3%), output (1.1%) and, ultimately, consumption (0.6%). On the top of that, shutting down the reorganization chapter when Ch 7 is efficient is less costly.

The last exercise, addresses a long lasted question in the corporate bankruptcy literature: what is the optimal level of creditors' rights? The model suggests that, from a general equilibrium point of view an increase in creditors' rights - modelled as a decrease in the bargaining power of workers - is only beneficial. A 10% decrease in the firms' bargaining power, increases output by 0.9%.

The second chapter of this manuscript challenges this last result. In the paper "Bankruptcy reforms when workers extract rents" I show that an increase in creditor rights ought not to increase recovery values when workers extract rents.

By and large the paper highlights sizeable general equilibrium implications of changes in the corporate bankruptcy law design.

Appendix

1.A Appendix: Proof of Theorem 1.4.1

The proof is organized as follows[45]. First, I define the space $\mathscr{V}$ and show that is a complete metric space (Lemma 1.A.1). Second, I show that (Tv) maps $\mathscr{V}$ into $\mathscr{V}$ (Lemma 1.A.2). Third, I show that (Tv) is a contraction mapping in $(\mathscr{V}, \|\cdot\|)$ (Lemma). Then, from the contraction mapping theorem, the result follows (Theorem 1.4.1).

Since (k, z) is drawn at entry and constant for incumbent firms, the proof will proceed for a given $(k, z) \in \mathrm{K} \times \mathrm{Z}$. For notational simplicity, I will omit the dependence on (k, z).

1.A.1 $\mathscr{V}$ is a complete metric space

Proof. Let $\mathscr{V}$ define the space of continuous vector-valued function $v : \mathrm{X} \times \mathrm{K} \times Z \to \mathbb{R}^{|\mathrm{B}|}$ with the following properties:

A1

$$v \in [v_{\min}, v_{\max}]^{|\mathrm{B}|} = \Bigg[\frac{1}{1-\beta} \cdot \iota_I \cdot \min\left\{\left[\pi(x_{\min}) - \chi_o - \delta k - b_{\max} + \frac{1}{1+r} b_{\min}\right], 0\right\}, \\ \frac{1}{1-\beta}\left[\pi(x_{\max}) - \chi_o - \delta k - b_{\min} + q_{b_{\max}}(x_{\max}) b_{\max}\right]\Bigg]^{|\mathrm{B}|}$$

where I use the fact that $q_{b_{\min}}(x) = \dfrac{1}{1+r}\ \forall x \in \mathrm{X}$, $0 \leq q_{b_{\max}}(x_{\max}) \leq \dfrac{1}{1+r}$, $b_{\min} \in \mathrm{B}_-$ and $b_{\max} \in \mathrm{B}_+$.

A2 Let $b_0 < b_1$ then $v_{b_0}(x) \geq v_{b_1}(x)$

A3 Let $x_0 < x_1$ then $v_b(x_0) \leq v_b(x_1)$

Lemma 1.A.1. *$\mathscr{V}$ is a not empty, complete metric space endowed with the metric:*

$$\|v\| = \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b(x)|$$

Proof. The proof proceeds in steps.

- $\mathscr{V}$ **is not empty**. Pick any constant vector-valued function $\bar{v} \in \mathbb{R}^{|\mathrm{B}|}$ such that $\bar{v} \in [v_{min}, v_{max}]^{|\mathrm{B}|}$. $\bar{v}$ trivially satisfies the monotonicity conditions A2-A3. Hence $\mathscr{V}$ is not empty.

[45] The proof draws on Chatterjee et al. [2007].

- $\mathscr{V}$ **is a metric space**. Let $v^1, v^2, v^3 \in \mathscr{V}$ and $\rho : \mathscr{V} \times \mathscr{V} \to \mathbb{R}$ with $\rho(v_1, v_2) = \|v_1 - v_2\|$. Then:
i) $\rho(v^1, v^1) = \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^1(x)| = 0$
ii) $\rho(v^1, v^2) = \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^2(x)| > 0$
iii)
$$\begin{aligned}\rho(v^1, v^2) &= \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^2(x)| \\ &= \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^2(x) - v_b^1(x)| = \rho(v^2, v^1)\end{aligned}$$
iv)
$$\begin{aligned}\rho(v^1, v^3) &= \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^3(x)| \\ &= \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^2(x) + v_b^2(x) - v_b^3(x)| \\ &\leq \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^2(x)| + |v_b^2(x) - v_b^3(x)| \\ &\leq \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^1(x) - v_b^2(x)| \\ &\quad + \max_{b \in \mathrm{B}} \sup_{x \in \mathrm{X}} |v_b^2(x) - v_b^3(x)|\end{aligned}$$

- $\mathscr{V}$ **is a complete metric space**. Let $\mathscr{C}$ the set of continuous and bounded vector-valued functions $f : \mathrm{X} \to \mathbb{R}^{|\mathrm{B}|}$. Then in order to prove that $\mathscr{V}$ is a complete metric space is sufficient to prove that $\mathscr{V}$ is a closed subset of $\mathscr{C}$, and then apply (an extension of) the Exercise 3.6.b of Stokey et al. [1990a]. In order to prove that $\mathscr{V}$ is closed we will prove that $\forall \{v_m\} \in \mathscr{V}$, with $v_m \to_{\|\cdot\|} v^*$, then v^* belongs to $\mathscr{V}$. Let $\{v^n\} \in \mathscr{V}$, hence each term v^n of the sequence of vectors is continuous on X, and satisfies A1-A3.
    - v^* *satisfies Properties A1-A3.* Since by A1 v^n is bounded, then by Bolzano-Weirstrass Theorem there exists a uniformly (on X) convergent subsequence $v^{n_k} \to_{\|\cdot\|} v^*$, which trivially satisfies A1 (boundedness). Let $b_0 < b_1$. Since v^n satisfies A2 then: $v_{b_0}^{n_k}(x) \geq v_{b_1}^{n_k}(x)$. Since weak inequalities are preserved under the limit $v_{b_0}^*(x) \geq v_{b_1}^*(x)$, and v^* satisfies A2. With a similar reasoning we can prove that v^* satisfies the other monotonicity property A3.
    - v^* *is continuous on* X. v^* is continuous if all its |B| components are continuous. Hence the proof proceeds component-wise.

Fix $b \in B$. Let

$$\|v_b\|_{\mathrm{X}} = \sup_{x \in \mathrm{X}} |v_b(x)|$$

Pick a cauchy sequence $\{v_b\} \in \mathscr{V}$. Then for any given $x \in \mathrm{X}$, let us look at the sequence of real numbers $\{v_b^{n_k}(x)\}$ (evaluated at x). Then

$$|v_b^n(x) - v_b^m(x)| \leq \sup_{x \in \mathrm{X}} |v_b^n(x) - v_b^m(x)| \equiv \|v_b^n - v_b^m\|_{\mathrm{X}}$$

Since, by assumption $\{v_b\}$ is Cauchy, then $\{v_b^{n_k}(x)\}$ is a cauchy sequence of real numbers. By completeness of real numbers, $\{v_b^{n_k}(x)\}$ converges to a limit point, say, $v_b^*(x)$, $v_b^{n_k}(x) \to v_b^*(x)$. We need to prove $v_b^{n_k} \to_{\|\cdot\|_{\mathrm{X}}} v_b^*$. Fix $\epsilon > 0$. Since $\{v_b\}$ is a cauchy sequence, then there exists an $N_1(\epsilon)$ such that $\forall m, n \geq N_1(\epsilon)$[46], we have that $|v_b^n(x) - v_b^m(x)| \leq \frac{\epsilon}{2}$, or equivalently $\|v_b^n - v_b^m\|_{\mathrm{X}} \leq \frac{\epsilon}{2}$. Now for any fixed $x \in \mathrm{X}$ and $\forall m, n \geq N_1(\epsilon)$,

$$\begin{aligned}
|v_b^n(x) - v_b^*(x)| &= |v_b^n(x) - v_b^m(x) + v_b^m(x) - v_b^*(x)| \\
&\leq \|v_b^n - v_b^m\| + |v_b^m(x) - v_b^*(x)| \\
&\leq \frac{\epsilon}{2} + |v_b^m(x) - v_b^*(x)|
\end{aligned}$$

Since $v_b^m(x) \to v_b^*(x)$, for any given x there exists an $N_2(\epsilon, x)$, such that $\forall m > N_2(\epsilon, x)$, we have $|v_b^m(x) - v_b^*(x)| \leq \frac{\epsilon}{2}$[47]. Since x was arbitrary, there exists an $N_2(\epsilon)$, independent of x, such that $\forall m > N_2(\epsilon)$, we have $\forall x \; v_b^m(x) \to v_b^*(x)$. Hence $\|v_b^n - v_b^*\|_{\mathrm{X}} \leq \epsilon$, $\forall n > N_1(\epsilon)$. Since ϵ was arbitrary, the result follows.
Now, $\forall \epsilon > 0$ there exists an $N(\epsilon)$, such that $\forall n_k > N(\epsilon)$ $\|v_b^{n_k} - v_b^*\|_{\mathrm{X}} < \frac{\epsilon}{3}$.
Since $\{v_b^{n_k}\}$ is a sequence of continuous function then $\forall \epsilon > 0$ there exists a $\delta(\epsilon)$ such that $\forall x, y \in \mathrm{X}$ with $\|x - y\|_E < \delta(\epsilon)$[48] then $|v_b^{n_k}(x) - v_b^{n_k}(y)| < \frac{\epsilon}{3}$.
Hence:

$$\begin{aligned}
|v_b^*(x) - v_b^*(y)| &= |v_b^*(x) - v_b^{n_k}(x) + v_b^{n_k}(x) - v_b^{n_k}(y) + v_b^{n_k}(y) - v_b^*(y)| \\
&\leq |v_b^*(x) - v_b^{n_k}(x)| + |v_b^{n_k}(x) - v_b^{n_k}(y)| + |v_b^{n_k}(y) - v_b^*(y)| \\
&\leq \|v_b^* - v_b^{n_k}\|_{\mathrm{X}} + |v_b^{n_k}(x) - v_b^{n_k}(y)| + \|v_b^{n_k} - v_b^*\|_{\mathrm{X}} < \epsilon
\end{aligned}$$

Since b was arbitrary, then v^* is continuous on all its components $b \in \mathrm{B}$, and therefore is continuous.

[46]Notice $N_1(\epsilon)$ does not depend on x.
[47]Notice the weak inequality is preserved when in the RHS we make $v_b^m(x) \to v_b^*(x)$.
[48]Where $\|\cdot\|_E$ denote the Euclidean norm in the normed vector space S (Euclidean Space).

It follows that $\mathscr{V}$ is a complete metric space. □

1.A.2 $T : \mathscr{V} \to \mathscr{V}$

Lemma 1.A.2.1. (Tv) **is continuous. on** $\mathrm{X} \times \mathrm{B}$

Proof. Since a vector-valued function is continuous if and only if all its components are continuous, the proof proceeds component-wise, $\forall b \in \mathrm{B}$. Given $(k, z) \in \mathrm{K} \times \mathrm{Z}$ the firm value under liquidation $V^L(\cdot)$ and exit $V^X(\cdot)$ are constant over X and therefore trivially continuous. Hence, we just need to prove continuity of $V^C(\cdot)$ (value under continuation) and $V^R(\cdot)$ (value under reorganization). Since if $b \in \mathrm{B}_-$ (savings) $V^R(\cdot)$ is not defined, w.l.o.g. I will assume that $b \in \mathrm{B}_-$ when considering $\phi_X, \phi_D, \phi_R = (1, 1, 1)$.

Let me now introduce some notations. Let $(Pv)(x) = \{(Pv)(b', x) : b' \in B\}$, denote the Markov operator

$$(Pv)^{b'}(x) = \int_{\mathrm{X}} v_{b'}(x')Q(x, \mathrm{d}x') = (Pv)^{b'}(x) \tag{1.A.1}$$

Given (ϕ_X, ϕ_D, ϕ_R) let

$$F_b^{b',\phi_X,\phi_D,\phi_R}(x) = \begin{cases} g\left(\pi(x) - \chi_o + q_{b'}(x)b' - \delta k - b\right) & (\phi_X, \phi_D, \phi_R) = (0, 0, 0) \\ \iota \cdot \left[\pi(x) - \chi_o - \chi_b + q_{b'}(x)b' - \delta k - \alpha_b(x) \cdot b\right] & (\phi_X, \phi_D, \phi_R) = (1, 1, 1) \end{cases}$$

be the return function, where I used the fact that firm's preferences are monotonic in d to set the constraints in the continuation problem 1.10 and reorganization problem 1.12 binding. Then I can define the objective function

$$\omega_b^{b',\phi_X,\phi_D,\phi_R}(x) = F_b^{b',\phi_X,\phi_D,\phi_R}(x) + \beta(Pv)^{b'}(x)$$

Now, let

$$W_{b,\phi_X,\phi_D,\phi_R}(x) = \left[\pi(x) - \chi_o - \phi_X, \phi_D\phi_R\chi_b - \delta k - (1 - \phi_X)b - \phi_X\phi_D\phi_R\alpha_b^*(x)b\right] \tag{1.A.2}$$

denote the net wealth of a firm. Then we can rewrite the budget correspondence

$$B_{b,\phi_X,\phi_D,\phi_R}(x) = \begin{cases} \left\{(d, b') \in \mathrm{D} \times \mathrm{B} : d - q_{b'}(x)b' \leq W_{b,\phi_X,\phi_D,\phi_R}(x)\right\} & (\phi_X, \phi_D, \phi_R) = (0, 0, 0) \\ \left\{(d, b') \in \mathrm{D}_- \times \mathrm{B}_- : d - q_{b'}(x)b' \leq W_{b,\phi_X,\phi_D,\phi_R}(x)\right\} & (\phi_X, \phi_D, \phi_R) = (1, 1, 1) \end{cases} \tag{1.A.3}$$

or more compactly

$$B_{b,\phi_X,\phi_D,\phi_R}(x) = \begin{cases} \left\{(d, b') \in \mathrm{D} \times \mathrm{B} : H_{b,\phi_X,\phi_D,\phi_R}(x, d, b') \geq 0\right\} & (\phi_X, \phi_D, \phi_R) = (0, 0, 0) \\ \left\{(d, b') \in \mathrm{D}_- \times \mathrm{B}_- : H_{b,\phi_X,\phi_D,\phi_R}(x, d, b') \geq 0\right\} & (\phi_X, \phi_D, \phi_R) = (1, 1, 1) \end{cases} \tag{1.A.4}$$

where

$$H_{b,\phi_X,\phi_D,\phi_R}(x,d,b') = (1-\phi_X+\phi_X\phi_D\phi_R)\cdot\left[W_{b,\phi_X,\phi_D,\phi_R}(x)+q_{b'}(x)b'-d\right] \tag{1.A.5}$$

Ultimately let $p(b',x) = [1,-q_{b'}(x)]$. Then, given $y=[d,b']'$, I can define the separating hyperplane

$$B_{b,\phi_X,\phi_D,\phi_R}(x) = \begin{cases} \left\{y\in D\times B : p(b',x)\cdot y \leq W_{b,\phi_X,\phi_D,\phi_R}(x)\right\} & (\phi_X,\phi_D,\phi_R)=(0,0,0)\\ \left\{y\in \mathrm{D}_-\times \mathrm{B}_- : p(b',x)\cdot y \leq W_{b,\phi_X,\phi_D,\phi_R}(x)\right\} & (\phi_X,\phi_D,\phi_R)=(1,1,1)\\ \left\{y\in \mathrm{D}\times\{0\} : p(b',x)\cdot y \leq (1-\delta)\cdot k-b\right\} & (\phi_X,\phi_D,\phi_R)=(1,0,0)\end{cases} \tag{1.A.6}$$

The proof is organized as follows. First, I show that $V^C(\cdot)$ and $V^R(\cdot)$ are continuous on X by Berge's maximum theorem. Then, since the maximum of two continuous function is continuous, $(Tv)(b,x;k,z)$ is continuous on X for a given $b\in$ B. The proof concludes noticing that (Tv) is continuous on all its components, $b\in$ B.

1. *$V^C(\cdot)$ and $V^R(\cdot)$ are continuous on* X *(by Berge's maximum theorem).* Given $(\phi_X,\phi_D,\phi_R)\in\{(0,0,0),(1,1,1)\}$: 1) I show that the objective function $\omega_b^{b',\phi_X,\phi_D,\phi_R}(x)$ is continuous on $(x,b')\in$ X × B for $(\phi_X,\phi_D,\phi_R)\in\{(0,0,0),(1,1,1)\}$; 2) I show that the correspondence $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is not empty, compact-valued and continuous.

   By Berge's Maximum theorem, it follows that (Tv) is continuous on X.

   - *1.1. The objective function $\omega_b^{b',\phi_X,\phi_D,\phi_R}(x)$ is continuous on (b',x).*

     Proof. Since B is discrete, $\omega_b^{b',\phi_X,\phi_D,\phi_R}(d,x)$ is trivially continuous on b'. It remains to prove that $\omega_b^{b',\phi_X,\phi_D,\phi_R}(d,x)$ is continuous on x.
     Since $v\in\mathscr{V}$ and $Q(x,\mathrm{d}x')$ satisfies the strong Feller Property[49], then the Markov operator (Pv) is continuous on $x\in$ X.
     By Theorem (1.4.2) $\alpha_b(x)$ is continuous on $x\in$ X. By inspection,

$$\begin{aligned} q_{b'}(x)b' &= R^7(k)\cdot\int_{\mathrm{X}} I_{\phi_X,\phi_D,\phi_R=(1,1,0)}Q(x,\mathrm{d}x') && \textit{Ch 7 default}\\ &+\int_{\mathrm{X}}\alpha_{b'}(x')b'I_{\phi_X,\phi_D,\phi_R=(1,1,1)}Q(x,\mathrm{d}x') && \textit{Ch 11 default}\\ &+b'\cdot\int_{\mathrm{X}}\left[I_{\phi_X,\phi_D,\phi_R=(0,0,0)}+I_{\phi_X,\phi_D,\phi_R=(1,0,0)}\right]Q(x,\mathrm{d}x') && \textit{Cont. / Exit}\end{aligned}$$

[49] The stochastic kernel $Q(x,\mathrm{d}x')$ is strongly continuous (satisfies the *strong Feller property*) if $\int v(x')Q(x,\mathrm{d}x')\in\mathscr{C}_b(\mathrm{X})$ for any $v\in M_b(\mathrm{X})$, space of bounded functions (Onésimo and Lasserre [1996]).

$q_{b'}(x)b'$ is continuous on x, since v is continuous, and $Q(x, \mathrm{d}x')$ is continuous on x and satisfies the *strong* Feller property ($Q(x, \mathrm{dx}')$ is required to be *strongly* continuous). As a result $F_b^{b',\phi_X,\phi_D,\phi_R}(x)$ is continuous on (x, b').

Then, the objective function $\omega_b^{b',\phi_X,\phi_D,\phi_R}(x)$ is **continuous** on $(x, b') \in \mathrm{X} \times \mathrm{B}$. □

- *1.2. $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is not empty, compact-valued and continuous.*

Proof. Since $(b', d) = (0, 0) \in B_{b,\phi_X,\phi_D,\phi_R}(x)$ for any $x \in \mathrm{X}$, then $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is **not empty-valued**.
Since X is a compact set and $\mathrm{D} \times \mathrm{B}$ is compact, the graph $B^{\text{graph}}_{b,\phi_X,\phi_D,\phi_R}(x)$ of $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is bounded, and therefore $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is **bounded**.
Given (ϕ_X, ϕ_D, ϕ_R), Since $\alpha_b^*(\cdot)$, $q(\cdot)$ are continuous on x we have that $H(\cdot)$ (equation (1.A.5)) is continuous on (x, d, b'). Let $\{x_n\} \to x$ a convergent subsequence (which exists by the Bolzano-Weirstrass theorem). Since $B_{b,\phi_X,\phi_D,\phi_R}(\cdot)$ is not empty-valued, let us pick a sequence $\{(b'_n, d_n)\}$, such that $(b'_n, d_n) \in B_{b,\phi_X,\phi_D,\phi_R}(x_n)\ \forall n$. Since $\{x_n\} \to x$ there is a bounded set $\hat{\mathrm{X}} \subseteq \mathrm{X}$ which contains $\{x_n\}$ and x. Since $B_{b,\phi_X,\phi_D,\phi_R}(\hat{\mathrm{X}})$ is bounded, $\{(b'_n, d_n)\}$ has a convergent subsequence $\{(b'_{n_k}, d_{n_k})\} \to (b', d)$.
Since $H(x_{n_k}, b'_{n_k}, d_{n_k}) \geq 0$ for all n_k, by continuity and the fact that the limit preserves weak inequalities $H(x, b', d) \geq 0$ and therefore $(b', d) \in B_{b,\phi_X,\phi_D,\phi_R}(x)$.
Since x was arbitrary, $B^{\text{graph}}_{b,\phi_X,\phi_D,\phi_R}(x)$ is closed and therefore $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is **compact-valued**. Then since $\{(b'_n, d_n)\}$ was arbitrary, $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is **uhc**.
The proof that $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is lhc on X follows by direct application of Theorem 1.2 in Harris [1987] p. 12[50]. First for a given $b' \in \mathrm{B}$, let

$$B^{b'}_{b,\phi_X,\phi_D,\phi_R}(x) = \begin{cases} \{d \in \mathrm{D} : d \leq f(x)\} & (\phi_X, \phi_D, \phi_R) = (0,0,0) \\ \{d \in \mathrm{D}_- : d \leq f(x)\} & (\phi_X, \phi_D, \phi_R) = (1,1,1) \end{cases} \tag{1.A.7}$$

be the projection of $B_{b,\phi_X,\phi_D,\phi_R}(x)$ on X for some $b' \in \mathrm{B}$, where

$$f(x) \equiv W_{b,\phi_X,\phi_D,\phi_R}(x) + q_{b'}(x)b'$$

The result follows by realizing that $f(x)$ is continuous in x, and that $b' \in \mathrm{B}$ was arbitrary.
It follows that $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is continuous on X. □

[50] *Suppose $f : X \to Y$ is continuous on X and Y is compact subset of $\mathbb{R}$. Define $\Gamma : X \to 2^Y$ by $\Gamma(x) = \{y \in Y : y \leq f(x)\}$. Then Γ is continuous on X* (Harris [1987]).

Since for any $b \in \mathrm{B}$, $(\phi_X, \phi_D, \phi_R) \in \{(0,0,0),(1,1,1)\}$, $\omega_b^{b',\phi_X,\phi_D,\phi_R}(x)$ is continuous on $\mathrm{X} \times \mathrm{B} \times \mathrm{D} \times \mathrm{B}$ and $B_{b,\phi_X,\phi_D,\phi_R}(x)$ is a not empty, compact-valued and continuous correspondence, by Berge's maximum theorem, $V^C(\cdot)$ and $V^R(\cdot)$ are continuous on X.

2. $\max\limits_{\phi_X}\left\{V^C(\cdot), \max\limits_{\phi_D}\left\{V^X, \max\limits_{\phi_R}\{V^R(\cdot), 0\}\right\}\right\}$ *is continuous.*

Proof. Let $m(x) = \max\{f(x), g(x)\}$. Fix x_0. If w.l.o.g $f(x_0) > g(x_0)$ then $m(x_0) = f(x_0)$ and the continuity of $m(\cdot)$ on x_0 follows trivially. Let us focus on the interesting case where $f(x_0) = g(x_0)$. Take $\epsilon > 0$, then there is a δ_f such that $|f(x) - f(x_0)| < \epsilon$ for $|x - x_0| < \delta_f$. Similarly there is a δ_g such that $|g(x) - g(x_0)| < \epsilon$ for $|x - x_0| < \delta_g$. Let $\delta = \min\{\delta_f, \delta_g\}$. Since $m(x_0) = f(x_0) = g(x_0)$ we have that $|m(x) - m(x_0)| < \epsilon$ whether $m(x) = f(x)$ or $m(x) = g(x)$ as long as $|x - x_0| < \delta$. Since x_0 was arbitrary, iterating the procedure the result follows. □

Since $b \in \mathrm{B}$ was arbitrary, then it follows that (Tv) is continuous. □

Lemma 1.A.2.3. (Tv) satisfies A1-A3.
In this section I prove the boundedness (A1) and monotonicity properties (A2-A3) of (Tv). In the proof of the monotonicity properties I use the fact that a maximum over a smaller set is smaller than the one on a larger set.
Lemma 1.A.2.2. (Tv) satisfies A1 (boundedness property).

Proof. The minimum dividend is given by $\underline{d} = \min\left\{\pi(x_{\min}) - \chi_o - \delta k - b_{\max} + \frac{1}{1+r} b_{\min}, 0\right\}$ and the maximum dividend is given by $\bar{d} = \pi(x_{\max}) - \chi_o - \delta k - b_{\min} + q_{b_{\max}}(x_{\max}) b_{\max}$
Then $(Tv) \in [\frac{1}{1-\beta} \cdot \iota_I \cdot \underline{d}, \frac{1}{1-\beta} \cdot \bar{d}]^{|\mathrm{B}|} = [v_{\min}, v_{\max}]^{|\mathrm{B}|}$. □

(Tv) **satisfies A2**. Let $b_0 < b_1$. Then $(Tv)(b_0) \geq (Tv)(b_1)$.

Proof. Assume $(\phi_X, \phi_D, \phi_R) = (0,0,0)$. It is easy to see that $W_{b_1,0,0,0} < W_{b_0,0,0,0}$, and therefore that $B_{b_1,0,0,0} \subseteq B_{b_0,0,0,0}$. The result follows. Assume $(\phi_X, \phi_D, \phi_R) = (1,1,1)$. By theorem 1.5.1, $\alpha_b^* b$ is independent on b. Hence $B_{b_1,1,1,1} = B_{b_0,1,1,1}$, and the result follows. If $(\phi_X, \phi_D, \phi_R) = (1,1,0)$ than $B_{b_1,1,1,0}(x) = B_{b_0,1,1,0}(x) = \varnothing$ and the result trivially follows. If $(\phi_X, \phi_D, \phi_R) = (1,0,0)$ than $B_{b_1,1,0,0}(x) \subset B_{b_0,1,0,0}(x)$ and the result follows. □

(Tv) **satisfies A3**. Let $x_0 < x_1$. Then: $(Tv)(x_0) \leq (Tv)(x_1)$.
Let $B_{b,\phi_X,\phi_D,\phi_R}(x) \equiv B\left(p(b', x), W_{b,\phi_X,\phi_D,\phi_R}(x)\right)$. and let us establish the following result

$$B\left(p(b', x_0), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) \subseteq B\left(p(b', x_1), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) \tag{1.A.8}$$

Proof. For any feasible choice vector $y = (b', d)$ we have $p(b', x_1)y \leq p(b', x_0)y$. The result follows from the fact that In particular

$$\begin{aligned} p(b', x_1)y &= d - q_{b'}(x_1)b' \\ &= d - q_{b'}(x_1)b' + [q_{b'}(x_0)b' - q_{b'}(x_0)b'] \\ &= d - q_{b'}(x_0)b' - [q_{b'}(x_1)b' - q_{b'}(x_0)b'] \\ &= d - q_{b'}(x_0)b' - \Delta_q = p(b', x_0) - \Delta_q \end{aligned}$$

and that, $\Delta_q = [q_{b'}(x_1)b' - q_{b'}(x_0)b'] \geq 0$ by monotonicity properties of the transition function $Q(x, \mathrm{d}x')$. Then:

$$\begin{aligned} B\left(p(b', x_1), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) &= \{y \in \mathbb{R}^2 : p(b', x_1)y \leq W_{b,\phi_X,\phi_D,\phi_R}(x_0)\} \\ &= \{y \in \mathbb{R}^2 : p(b', x_0)y \leq W_{b,\phi_X,\phi_D,\phi_R}(x_0) + \Delta_q\} \\ &= B\left(p(b', x_0), W_{b,\phi_X,\phi_D,\phi_R}(x_0) + \Delta_q\right) \end{aligned} \tag{1.A.9}$$

Then

$$B\left(p(x_0), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) \subseteq B\left(p(x_0), W_{b,\phi_X,\phi_D,\phi_R}(x_0) + \Delta_q\right) = B\left(p(x_1), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right)$$

where in the first inclusion I use the fact that $W_{b,\phi_X,\phi_D,\phi_R}(x_0) < W_{b,\phi_X,\phi_D,\phi_R}(x_0) + \Delta_q$, and where the last equality comes from (1.A.9). The result follows. □

Proof. Assume $(\phi_X, \phi_D, \phi_R) \in \{(0, 0, 0), (1, 1, 1)\}$. It is easy to see that $W_{b,\phi_X,\phi_D,\phi_R}(x_0) < W_{b,\phi_X,\phi_D,\phi_R}(x_1)$. Then

$$\begin{aligned} B_{\phi_X,\phi_D,\phi_R}(x_0) \equiv & B\left(p(b', x_0), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) \subset B\left(p(b', x_1), W_{b,\phi_X,\phi_D,\phi_R}(x_0)\right) \\ \subset & B\left(p(b', x_1), W_{b,\phi_X,\phi_D,\phi_R}(x_1)\right) \equiv B_{\phi_X,\phi_D,\phi_R}(x_1) \end{aligned}$$

where the first inclusion comes from (1.A.8). The result follows. If $(\phi_X, \phi_D, \phi_R) = (1, 1, 0)$ than $B_{b,1,1,0}(x_0) = B_{b,1,1,0}(x_1) = \varnothing$ and the result trivially follows. If $(\phi_X, \phi_D, \phi_R) = (1, 0, 0)$ than $B_{b,1,0,0}(x)$ is independent on x and the result follows.

□

Lemma 1.A.2. $T : \mathscr{V} \to \mathscr{V}$.

Proof. Putting together **Lemma 1.A.2.1. Lemma 1.A.2.2.** and **Lemma 1.A.2.3.** we have that $T : \mathscr{V} \to \mathscr{V}$. □

1.A.3 T is a contraction mapping with modulus β in $(\mathscr{V}, \|\cdot\|)$

Lemma 1.A.3. T satisfies monotonicity and discounting

Lemma 1.A.3.1. Monotonicity.

Proof. Let $b' \in B$, let $f, g \in \mathscr{V}$ with $f \leq g$. Then, by Monotonicity property of the Lebesgue integral

$$\int_X f_{b'}(x')Q(x, \mathrm{d}x') \leq \int_X g_{b'}(x')Q(x, \mathrm{d}x')$$

Since this result holds $\forall\ b' \in B$, then $(Tf)(b, x) \leq (Tg)(b, x)$. $\square$

Lemma 1.A.3.2. Discounting.

Proof. Let $a \in R$. Then:

$$\begin{aligned}
&\beta \int_X (f_{b'}(x') + a)Q(x, \mathrm{d}x') \\
&= \beta \int_X (f_{b'}(x')Q(x, \mathrm{d}x') + \beta a \int_X Q(x, \mathrm{d}x') \\
&= \beta \int_X (f_{b'}(x')Q(x, \mathrm{d}x') + \beta a
\end{aligned}$$

where in the second line I used the linearity property of the Lebesgue integral, and in the third line the fact that by definition of transition matrix $Q(x, \cdot)$ is a probability measure. Then:

$$\begin{aligned}
(Tv)(b, x) &= \max_{\phi_X} \Bigg\{ \max_{(d,b')\in B_{0,0,0}} g(d) + \beta \int_X (f_{b'}(x') + a)Q(x, \mathrm{d}x')\,, \\
&\max_{\phi_D} \Bigg[V^X\,,\ \max_{\phi_R} \Bigg\{ \max_{(d,b',k)\in B_{1,1,1}} g(d) + \beta \int_X (f_{b'}(x') + a)Q(x, \mathrm{d}x')\,,\ 0 \Bigg\} \Bigg] \Bigg\} \\
&= \max_{\phi_X} \Bigg\{ \max_{(d,b')\in B_{0,0,0}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x') + \beta a\,, \\
&\max_{\phi_D} \Bigg[V^X\,,\ \max_{\phi_R} \Bigg\{ \max_{(d,b',k)\in B_{1,1,1}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x') + \beta a\,,\ 0 \Bigg\} \Bigg] \Bigg\} \\
&\leq \max_{\phi_X} \Bigg\{ \max_{(d,b')\in B_{0,0,0}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x') + \beta a\,, \\
&\max_{\phi_D} \Bigg[V^X + \beta a\,,\ \max_{\phi_R} \Bigg\{ \max_{(d,b',k)\in B_{1,1,1}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x') + \beta a\,,\ \beta a \Bigg\} \Bigg] \Bigg\} \\
&= \max_{\phi_X} \Bigg\{ \max_{(d,b')\in B_{0,0,0}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x')\,, \\
&\max_{\phi_D} \Bigg[V^X\,,\ \max_{\phi_R} \Bigg\{ \max_{(d,b',k)\in B_{1,1,1}} g(d) + \beta \int_X f_{b'}(x')Q(x, \mathrm{d}x')\,, \Bigg\} \Bigg] \Bigg\} + \beta a = \\
&= (Tv)(b, x) + \beta a
\end{aligned}$$

where the inequality comes from the fact that $\max\{A + \beta a, 0\} \leq \max\{A + \beta a, \beta a\}$. □

Lemma 1.A.4. *(Tv) is a contraction mapping with modulus β.*

Proof. (Tv) maps the space $\mathscr{V}$ in $\mathscr{V}$ (Lemma 1.A.2). Noticing $\mathscr{V} \subset B(\mathrm{X})$ (where $B(\mathrm{X})$ denotes the space of bounded function), and since by Lemma 1.A.3 (Tv) satisfies the monotonicity and discounting properties, then (Tv) is a contraction mapping. □

1.A.4 Theorem 1.4.1

Theorem 1.4.1 *There exists a unique $v^* \in \mathscr{V}$ such that: $v^* = (Tv^*)$. Moreover: i) v^* is continuous and bounded, and it satisfies the monotonicity properties A1-A3. ii) The optimal correspondence $B_b^*(x)$ is compact-valued and upper hemicontinuous.*

Proof. Since $\mathscr{V}$ is a complete metric space (Lemma 1.A.1), and $(Tv) : \mathscr{V} \to \mathscr{V}$ is a contraction mapping with modulus β (Lemma 1.A.4) applying the Contraction mapping theorem (Theorem 3.2, Stokey et al. [1990a]) there exists a unique $v^* = (Tv^*)$. *i)* follows from the properties of the space of function considered. The fact that $B_b^*(x)$ is compact and upper hemicontinuous is a direct consequence of Berge's Maximum Theorem. □

1.B Appendix: Proof of Theorem 1.4.2

Theorem 1.4.2. *There exists a unique $\alpha^* : \mathrm{X}\times\mathrm{K}\times\mathrm{Z} \to \mathrm{A}$, with $\alpha_b^*(x,k,z) \in \mathscr{C}_b^{[0,1]}(\mathrm{X}\times\mathrm{K}\times\mathrm{Z})$ $\forall b \in \mathrm{B}$, which solves* (1.12).

Since the surplus of the firm is monotonic in d, the constraint in (1.12) is binding. Since the two controls b' and a enter additively in the objective function and the derivative is a linear operator, the problems are separable. Hence we can rewrite the real (vector-)valued correspondence $\alpha^*(x) = \{\alpha_b^*(x,k,z) \in [0,1] : b \in \mathrm{B}\} \subset \mathbb{R}^{|\mathrm{B}|}$, with $\alpha^* : \mathrm{X}\times\mathrm{K}\times\mathrm{Z} \to \mathrm{A} \equiv [0,1]^{|\mathrm{B}|} \subset \mathbb{R}^{|\mathrm{B}|}$ defined in (1.12) as

$$\alpha_b^*(x,k,z) = \arg\max_{a\in[0,1]} \max_{b'\in\mathrm{B}} \left\{ S^f(a;b,x,k,z)^\theta \, S^c(a;b,x,k,z)^{1-\theta} \right\}$$

$$\text{s.t. } S^f(a;b,x,k,z) \geq 0, \quad S^c(a;b,x,k,z) \geq 0$$

Proof. The proof proceeds in 2 steps. Fix $b \in B$. Then:

1. *Claim.* For any $(x,k,z) \in \mathrm{X}\times\mathrm{K}\times\mathrm{Z}$, $\alpha_b^*(x,k,z) \subset \mathbb{R}$ is i) not-empty, ii) compact valued, iii) upperhemicontinuous.

Proof. Since $[0,1]$ is a not-empty, compact valued and continuous feasible correspondence, and the objective function is continuous (product of continuous functions), then by direct application of the Berge's Maximum Theorem the optimal correspondence α^* is not-empty compact-valued, uhc and it is contained in the feasible correspondence [0,1]. Noticing that x was arbitrary the result follows. □

2. *Claim.* Since $\theta \in \Theta \subset [0,1]$, then $\alpha_b^*(x,k,z) : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to [0,1]$ is a continous function.

   Proof. Since the objective function is strictly concave over the convex-compact valued set $[0,1]$, $\alpha_b^*(x,k,z)$ is unique (Lemma (1.D.1)). Since $\alpha_b^*(x,k,z)$ is uhc (Point 2) and single-valued, then $\alpha_b^*(x,k,z)$ is a continuous function. □

□

Lemma 1.B.1. *If* $\theta \in \Theta \subset [0,1]$, *if* $\alpha_b^*(x,k,z) : \mathrm{X} \times \mathrm{K} \times \mathrm{Z} \to [0,1]$ *which solves* (1.12) *exists then is unique,* $\forall b \in B$.

Proof.

$$\alpha_b^*(x,k,z) = \operatorname*{arg\,max}_{a\in[0,1]} \left\{ S^f(a;b,x,k,z)^{\theta}\, S^c(a;b,x,k,z)^{1-\theta} \right\} = \operatorname*{arg\,max}_{a\in[0,1]} NB(a;b,x,k,z)$$
$$\text{s.t. } S^f(a;b,x) \geq 0, \quad S^c(a;b,x,k,z) \geq 0$$

Let $NB(a;b,x,k,z) = S^f(a;b,x,k,z)^{\theta}\, S^c(a;b,x,k,z)^{1-\theta}$ denote the Nash Bargaining product as a function of a. $NB(a;b,x,k,z)$ is a continuous function (product of continuous function) over a compact support $[0,1]$. Hence by Berge's Maximum theorem the optimal correspondence $\alpha_b^*(x,k,z)$ is uhc.
Given (b,x,k,z), $NB(a;b,x,k,z)$ is twice continuously differentiable with respect to a. To show the result, we have to prove that $NB(a;b,x,k,z)$ is strictly concave over $a \in [0,1]$. By taking the second derivative of $NB(a;b,x,k,z)$ with respect to a

$$\begin{aligned}\frac{\partial^2 NB(a;b,x,k,z)}{\partial a^2} = &-\iota b\theta \underbrace{\left[(1-\theta)\iota b(S^f)^{\theta-2}(S^c)^{1-\theta} + b(S^f)^{\theta-1}(S^c)^{-\theta}\right]}_{+} \\ &- b(1-\theta)\underbrace{\left[\iota b(S^f)^{-\theta}(S^c)^{\theta-1} + \theta(S^f)^{\theta}(S^c)^{-\theta+1}\right]}_{+} < 0\end{aligned}$$

Hence for a given (b,x,k,z) there exists a unique solution $\alpha_b^*(x,k,z)$. Since (b,x,k,z) was arbitrary the result follows. □

1.C Appendix: Proof of Theorem 1.5.1

Let $\alpha^* \in \mathscr{C}_b^{[0,1]}(\mathrm{X}\times\mathrm{K}\times\mathrm{Z})$ *be the solution to the Nash Bargaining problem* (1.12). *Then for any* $(b,x,k,z)\in\mathrm{B}\times\mathrm{X}\times\mathrm{K}\times\mathrm{Z}$, *if the nash bargaining surplus*

$$S(x,k,z)\equiv \pi(x;k,z)-\delta k-(\chi_o+\chi_b)+q_{b^{*\prime}}(x,k,z)b^{*\prime}+\frac{\beta}{\iota}\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')-R^7(k)\geq 0$$

then an interior solution $\alpha_b^*(x,k,z)$ *exists and equals:*

$$\alpha_b^*(x,k,z)=\frac{R^7(k)}{b}+(1-\theta)\cdot\frac{S(x,k,z)}{b}$$

Proof. Let $NP(x,k,z)=\iota\cdot\left[\pi(x;k,z)-\delta k-(\chi_o+\chi_b)+q_{b^{*\prime}}(x,k,z)b^{*\prime}\right]$. For any $(b,x,k,z)\in B\times\mathrm{X}\times\mathrm{K}\times\mathrm{Z}$, taking the log of the objective function and taking partial derivative with respect to a

$$\theta\frac{\partial S^f(a;b,x,k,z)}{\partial a}\frac{1}{S^f(a;b,x,k,z)}+(1-\theta)\frac{b}{[ab-R^7(k)]}=0$$

$$\theta\frac{-\iota b}{S^f(a;b,x,k,z)}+(1-\theta)\frac{b}{[ab-R^7(k)]}=0$$

$$-\iota\theta\frac{1}{NP(x,k,z)-\iota ab+\beta\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')}+(1-\theta)\frac{1}{[ab-R^7(k)]}=0$$

$$\iota\theta[ab-R^7(k)]=(1-\theta)\left[NP(x,k,z)-\iota ab+\beta\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')\right]$$

$$ab=\frac{1}{\iota}\left[\iota\theta R^7(k)+(1-\theta)\left(NP(x,k,z)+\beta\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')\right)\right]$$

$$ab=\theta R^7(k)+(1-\theta)\left(\pi-\delta k-(\chi_o+\chi_b)+q_{b^{*\prime}}(x,k,z)b^{*\prime}+\frac{\beta}{\iota}\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')\right)$$

$$ab=R^7(k)+(1-\theta)\left(\pi-\delta k-(\chi_o+\chi_b)+q_{b^{*\prime}}(x,k,z)b^{*\prime}+\frac{\beta}{\iota}\cdot\int_{\mathrm{X}} v_{b^{*\prime}}(x';k,z)Q(x,\mathrm{d}x')-R^7(k)\right)$$

we get the interior solution:

$$\alpha_b^*(x,k,z)=\theta\frac{R^7(k)}{b}+(1-\theta)\cdot\frac{S(x,k,z)}{b}$$

□

1.D Appendix: Proof of Corollary 1.5.3

Proof. By variable transformation, let us define $\tilde{S^f}(a \cdot b; x, k, z) = S^f\,(a; b, x, k, z) \in \mathbb{R}$ and $\tilde{S^c}(a \cdot b; x, k, z) = S^c\,(a; b, x, k, z) \in \mathbb{R}$ as functions of $a \cdot b \in \mathrm{B}_+$.
Then I can rewrite the bargaining problem (1.12) as

$$\max_{a \cdot b \in \mathrm{B}_+} \left\{ \tilde{S^f}\,(a \cdot b; x, k, z)^{\theta}\, \tilde{S^c}\,(a \cdot b; x, k, z)^{1-\theta} \right\}, \quad \text{s.t. } \tilde{S^f}\,(a \cdot b; x, k, z) \geq 0, \quad \tilde{S^c}\,(a \cdot b; x, k, z) \geq 0$$

The (unique-)solution of the transformed problem $\alpha^* b > 0$ is function uniquely of (x, k, z) and does not depend on b, which concludes the proof. □

CHAPTER 2

Bankruptcy Reforms When Workers Extract Rents

2.1 Introduction

Do workers' rents matter for corporate bankruptcy reforms? The main argument in favour of pro-creditor bankruptcy reforms is that creditors recover more, and therefore lend more. This paper presents a novel channel through which this argument might break down: when workers extract rents, an increase in creditor rights ought not to increase *expected* recovery values. Pro-creditor bankruptcy reforms can actually backfire.

I start with the *theory*. Firms file for Chapter 11 (Ch 11) not only to restructure debt but *also* to restructure labour contracts[1]. From this observation, I build a theory where shareholders weigh the cost of restructuring labour contracts against their claims on the going-concern value of the firm. In this environment bankruptcy reforms face a trade-off. A more creditor-friendly bankruptcy law raises recovery values upon successful reorganizations, but at the expenses of the other stakeholders: workers and shareholders. Shareholders have less incentives to restructure labour contracts, making more likely that reorganizations fail and firms get liquidated[2]. *When* the drop in the likelihood of success of Ch 11 is larger than the increase in recovery values upon success, expected recovery values fall, the cost of debt rises and the bankruptcy reform backfires.

The bargaining power of workers determines *when* this happens. When workers do not extract rents, restructuring labour contracts does not affect the success of the reorganization. Conversely, when workers extract a lot of rents, failing to restructure labour contracts can prevent the firm from regaining economic soundness, and cause the reorganization to fail[3]. As a result, the effect of bankruptcy reforms on credit markets depends

[1]In U.S. corporations can ask for debt relief under Chapter 7 (Ch 7) - which disciplines the liquidation of the assets of the firm and ends with its dissolution - *and* Chapter 11 (Ch 11) - which disciplines the reorganization process among the stakeholders - bondholders, shareholders and *workers* - in the attempt of preserving the corporation as a going concern. Under Ch 11 a debtor is granted the possibility to reject *any* executory contract bondage that impairs the firm viability (§365 (a)). These contracts include debt obligations but, by far, do not reduce to them.

[2]By law, when a Chapter 11 reorganization fails, the case is converted to Chapter 7.

[3]The bankruptcy experiences of Delta Airlines and Hostess-Brands are illustrative in this sense. Both corporations filed for Ch 11 to reduce labour expenses, but the latter failed to find an agreement with the

on the bargaining power of workers.

To study the implications of bankruptcy reforms for economic activity, I propose a static model where resources are misallocated from their productive alternative because of an enforcement constraint, which I microfound using the corporate bankruptcy law. The expected recovery upon default determines the ex-ante lending, and the fraction of resources that are misallocated. I use this framework to answer analytically a normative question: what is the optimal level of creditor rights? For a given bargaining power of workers, the optimal (output maximizer) level of creditor rights weighs the benefit of higher recovery values upon successful reorganizations against a lower likelihood of success of the procedure. Since restructuring labour contracts matters more (for the success of Ch 11) as workers rents increase, the optimal level of creditor rights decreases with the bargaining power of workers.

The model illustrates how the bankruptcy law affects credit markets through the labour and debt restructuring activity, and how workers' rents alter this linkage. I call this channel the *restructuring channel*.

I turn to *facts*. In the late 90s the U.S. shifted towards a more pro-creditor bankruptcy reorganization process. I use this legal experiment to test the theory in the data. The empirical question is whether the shift in creditor rights protection affected in the same way firms facing different bargaining power of workers. To address it, I exploit two sources of variation: historical differences in the degree of unionization across states, and the change in the creditor rights protection regime. The analysis uses firm level accounting data from Compustat, bankruptcy information from UCLA LoPucki Bankruptcy Research Database , and a proxy for the bargaining power of workers from the Union Membership and Coverage database (CPS) , for the period 1979-2012.

I identify the break in the creditor rights protection regime in 1998 as the structural change in the relative use of the Ch 11 procedure with respect to the liquidation alternative, Ch 7. The break is associated with a drop in the likelihood of success of Ch 11 (from 95% to 92%), driven by highly unionized firms (from 96% to 92%). In the same spirit, Ch 11 becomes less attractive than Ch 7, especially for firms in highly unionized states. At the country level, firms experienced a dramatic deleveraging (−28%), mainly concentrated among highly unionized firms (−43%). Besides, dividend yields halved, and the Tobin-Q cross-sectional volatility tripled, again with significant differences across regions. Using regression techniques, I control for many sources of bias impairing the previous descriptive statistics analysis. The results hold through.

To study the positive implications of bankruptcy reforms, I build and characterize a general equilibrium dynamic model with heterogenous firms and default in equilibrium, where the default options capture salient features of the U.S. corporate bankruptcy law. I model the reorganization problem among the firms' stakeholders - shareholders, bond-

unions. While Delta airlines emerged on april 2007 with a 20% reduction in the employees and an healthier financial structure, Hostess Brand assisted at the piece-meal sale of its popular brands.

holders and workers - as a two stage Nash bargaining over labour and debt contracts, and a restructuring effort decision of the shareholders. Shareholders bargain first with workers and then with creditors. The rationale behind this assumption is *legal*. The U.S. Corporate bankruptcy law recognizes higher priority to workers over creditors on the firm surplus[4]. The consequence of this assumption is *economic*. Shareholders use the threat of liquidation in the second stage to reduce the bargaining position of workers in the first stage. The success of the reorganization process is stochastic and depends on the restructuring effort that shareholders decide to exert when bargaining with workers. In case of failure, the case is converted to Ch 7.

I conclude with *policy counterfactual experiments*. I calibrate the dynamic model to the U.S. economy from 1979 to 1998 to assess the macroeconomic and firm level implications of changes in the bankruptcy law. Then, I run two experiments.

In the first experiment, I examine the effects of the shift in creditor rights protection regime experienced by the U.S.. I discipline the increase in creditor rights to match the likelihood of success of Ch 11 in the post-break period. The model predicts changes in firms' bankruptcy choices and financial structure which are consistent with the data. In the model economy, the shift does not produce significant changes in aggregate TFP, output, and consumption, but has seizable regional effects: output increases in lowly unionized regions by 0.60% and decreases in highly unionized region by 0.36%. Consumption displays a similar behaviour.

In a second experiment, I try to attach a value to the bankruptcy reorganization procedure. The motivation rests on historical reasons. Ch 11 was introduced in 1978, with the enactment of the Corporate Bankruptcy Code. What if it had never been introduced? The model records a sizeable deleveraging (-20%) and drop in the dividend-price ratio (80%), driven by lowly unionized firms. Despite consumption and output would have barely changed in aggregate (−0.11%), the regional dynamics would have been dramatically different: output would have been 14% lower in highly unionized states, and 24% higher in lowly unionized ones.

The rest of the paper is organized as follows. Section 2.2 reviews the literature. Section 2.3 organizes the empirical evidence. Section 2.4 lays out the Static Model. Section 2.5 builds up the Dynamic Model. Section 2.6 performs the Quantitative Analysis, and Section 2.9 concludes.

[4]§507 (a) of the Bankruptcy Code.

2.2 Related Literature

This paper seats in the nexus between the macro-finance[5] and the corporate-bankruptcy literature. It contributes to recents macroeconomic studies of the interaction between the labour and credit market in presence of limited commitment[6], by foregrounding a novel mechanism within the lines[7] of the U.S. corporate bankruptcy law: the restructuring channel. As in these papers, the mechanism works through a collateral constraint, but its legislative hallmark allows me to study the effect of changes in the law on economic activity[8]. As in Biais and Mariotti [2009][9], the final effect of bankruptcy reforms depends on the interplay between the credit and labour market. In both frameworks pro-creditor bankruptcy reforms can backfire, but for different reasons[10]: while in Biais and Mariotti [2009] an increase in creditor rights yields higher expected recovery values (for a fixed wage), it ought not to be the case in my framework.

In this respect, the paper also departs from Corbae and D'Erasmo [2015][11] and Peri [2015][12] - which, in the spirit of Chatterjee, Corbae, Nakajima, and Rios-Rull [2007], firstly

[5]In particular, the macroeconomic literature that studies the impact of financial frictions on firm dynamics. Among others, Cooley and Quadrini [2001], Jermann and Quadrini [2008], Jermann and Quadrini [2012].

[6]Michelacci and Quadrini [2009] firstly studies the interaction between financial frictions and the labour markets in order to explain a few stylized facts about wages and firm dynamics. Hence Monacelli et al. [2011] looks at the business cycle implications of the presence of search friction under limited enforcement of debt contracts for the (un)employment fluctuations. In conclusion, Quadrini and Sun [2015] studies in a firm dynamic framework the use of capital structure policy to lower the hiring cost of the workers and provide empirical evidence of a strong correlation between hiring growh and debt growth, which increases with a proxy of the bargaining power of workers.

[7]In particular, §365 of Ch 11 of the U.S. corporate Bankruptcy law. Despite the restructuring of labour contract is a well established phenomenon in the bankruptcy literature (among others, Geva [2012]), it has not received the same attention by the economic literature.

[8]Hints on this nexus between bankruptcy and economic growth can already be found in La Porta et al. [1997, 1998], where financial development is partially proxied by variables that measure the efficiency of bankruptcy laws and the extent they protect the rights of the creditors. The seminal papers La Porta et al. [1997, 1998] pioneered the burgeoning literature that investigates the implications of institutions and regulations on the development of financial and credit markets. They find that better institutions and regulations are crucial to establish well-functioning financial and credit markets. Following, King and Levine [1993] and Rajan and Zingales [1998] document that improvements in the financial and credit markets foster economic growth, an idea that traces back to Schumpeter (1911).

[9]To the best of my knowledge, Biais and Mariotti [2009] represents the first attempt to study the general equilibrium implications of changes in the bankruptcy law, and the first attempt to illustrate how bankruptcy laws affect economic activity by altering the general equilibrium linkage between the credit market and labour market.

[10]In Biais and Mariotti [2009] theory a more pro-creditors bankruptcy law foster investment and labour demand, driving up wages and reducing profitability. This generates a trade-off, for which soft-laws can generate more utilitarian welfare than tough laws.

[11]Corbae and D'Erasmo [2015] investigates in a fully fledged firm dynamic model the macroeconomic implication of a longly debated bankruptcy reform suggested by Aghion et al. [1994] and recently proposed by the American Bankruptcy Institute. As a result, they document - among other aspects - an increase in consumption (2.32%), output (1.99%) and measured TFP (1.03%) after the adoption, due to cheaper borrowing, and better allocation of resources in the economy.

[12]Peri [2015] tries to identify the channels through which changes in the *actual* bankruptcy law design

investigate in a firm dynamic model á la Hopenhayn[13] the macroeconomic implications of changes in the U.S. corporate bankruptcy law - where an increase in the bargaining power of creditors yields higher recovery values. By breaking the identity between stakeholders[14] and stockholders[15] (enlarging the definition of stakeholders to include workers), and modelling the bargaining over labour contracts, expected recovery values can actually drop.

The focus on the misallocation of resources in the static model adds to the literature pioneered by Erosa and Cabrillana [2008]. As in Moll [2014][16], I investigate in a tractable framework the sources of misallocation in an economy where the severity of the financial frictions is pinned down by the court-supervised bankruptcy process and the bargaining power of workers.

The empirical analysis complements the findings of Klasa et al. [2009]and Matsa [2010][17] - which find that highly unionized firms tend to be more leveraged and to herd less on cash (than lowly unionized) - and the suggestive evidence of Quadrini and Sun [2015][18] - which find a positive correlation between employment growth and debt growth that is increasing in the level of unionization - by studying the effect of bankruptcy reforms on firms' bankruptcy choices and financial structure when workers extract rents.

In conclusion, I provide statistical support of the narrative approach followed by the bankruptcy literature - which has highlighted a shift in the creditor rights protection regime in the late 90s[19] - by identifying a break in the relative use of the Ch 7 and Ch 11 procedures by publicly listed firms in 1998.

2.3 Empirical Analysis

In the late 90s the U.S. shifted towards a more pro-creditor bankruptcy reorganization process. When did the shift in creditor rights protection regime happen? How did it affect

might affect TFP. Among others, I found that changes in the bankruptcy law that reduce the efficiency of liquidation affect TFP through the negative effect it exerts on large firms, despite large firms tend not to file for Ch 7. In doing so, I foreground a close relation between recovery value under Ch 11 and the equity issuance cost of the firm, which works through the forward looking nature of the debt restructuring process.

[13] This paper adds to the vast literature on firm dynamics pioneered by Hopenhayn [1992], in which the author extends the long run industry equilibrium theory by introducing a concept of stationary equilibrium, which allows him to investigate the phenomena of entry, exit and heterogeneity in the size and growth rates of firms.

[14] Stakeholders are formally defined as the agents that have an economic interest in the corporation.

[15] Namely, bondholders and shareholders

[16] Moll [2014] studies the effect of financial frictions on capital misallocation and aggregate productivity.

[17] Klasa et al. [2009] and Matsa [2010] study, respectively, the strategic use of corporate cash holdings and capital structure in collective bargaining with labor unions.

[18] Quadrini and Sun [2015] studies how workers extract rents affect firms' hiring choices in a dynamic model where firms use leverage to reduce the bargaining position of the workers at the cost of a higher likelihood of distress (*the bargaining channel*).

[19] Among others, Warren [1999] and Miller [2007].

the bankruptcy phenomenon and corporate structure of firms facing different bargaining power of workers?

I answer these questions by collecting firm level accounting data from Compustat North-America Fundamentals Annual , bankruptcy information from the UCLA LoPucki Bankruptcy Research Database and a proxy for the bargaining power of workers from the Union Membership and Coverage database (CPS)[20], for the period 1979-2012.

2.3.1 The Shift in the Creditor Rights Protection Regime

Ch 11 looks nowadays more creditor-friendly than it did 30 years ago[21]: when did the shift in creditor rights protection regime happen? Since an increase in creditor rights makes the reorganization procedure less attractive than its liquidation alternative, I identify the shift as the structural break in the relative use of the bankruptcy procedures. Two bankruptcy filing regimes emerge from the plot of annual filings for Ch 7 liquidation and Ch 11 reorganization by publicly traded firms from 1979 to 2012 (Fig. 2.3.1, Compustat data): while Ch 11 filings dominates Ch 7 filings in the pre-1998 period[22], the liquidation alternative is steadily more attractive in the ever after. The Quandt-likelihood-ratio test - for the presence of a structural break at an unknown date in the number of annual Ch 11 filings - corroborates the eye-ball inspection of a break in 1998 (Fig. 1.B.2).

The legal literature substantiates this finding (*narrative approach*). Warren [1999] and Miller [2007] explanations point at financial institutions lobbying for their bankruptcy agenda in 1997-1998[23].

2.3.2 Shift in Creditor Rights protection and the Bargaining Power of Workers

How did the shift in creditor rights affect firms facing different bargaining power of workers? To answer this question I exploit two sources of variation: historical differences in the

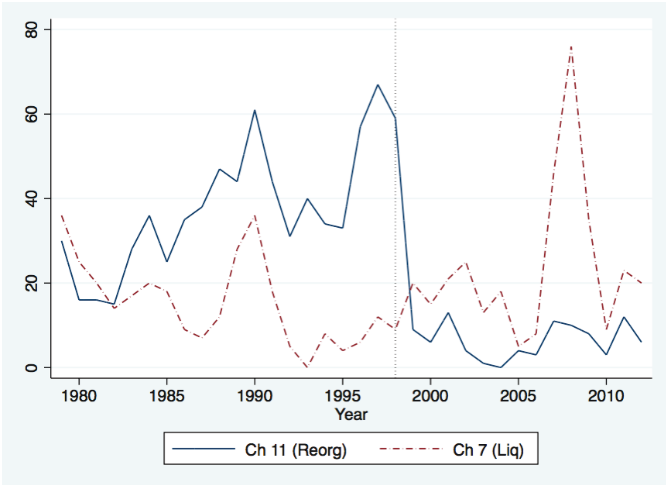
[20]The interested reader can refer to Appendix 1.A for a detailed description of the data.

[21]Baird and Rasmussen [2003], Ayotte and Morrison [2009] sustain that the reorganization process is now in the hands of the creditors. Warren and Westbrook [2003] pushes this argument further, by claiming that the debt-in-possession era has meet his end at the hands of the secured-party-in-possession era.

[22]With the exception of 1979-1980, Ch 11 filings always exceeds Ch 7 filings.

[23]In Warren [1999], professor at Harvard Law School Elizabeth Warren says and I quote '*According to the New York Times [K.Q. Seelye, "House to Vote Today on Legislation for Bankruptcy Overhaul" New York Times (9 June 1998) A18. 6.], financial institutions spent, in 1997 alone, about* 40 *million lobbying for their bankruptcy agenda - an amount matched only by the enormous tobacco lobby. One can only dream about how many millions were spent when lobbying intensified during 1998.*'. Miller [2007] reinforces this argument, arguing that these efforts have been channelled not only in the consumer bankruptcies but also on provisions related to Ch 11 reorganization. These provisions became statutory with the enactment of the Bankruptcy Abuse Prevention and Consumer Protection Act in 2004.

Figure 2.3.1: Annual Ch 7 and Ch 11 filings



Source: Compustat North-America Fundamentals Annual, 1950-2012. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

degree of unionization across states (*geographical dimension*), and the shift in the creditor rights protection regime (*time dimension*).

The U.S. Unionization Regions

I proxy the bargaining power of workers with the unionization coverage - the fraction of all employed civilian wage and salary workers covered by a collective bargaining agreement (Current Population Survey (CPS)) - of the U.S. state where the firm's headquarter is located. I then organize U.S. states in two regions based on their historical levels of unionization. I assign states to the highly [lowly] unionized region if their 1983-2014 average coverage is above [below] the median of the U.S. states 1983-2014 average coverages. Figure 2.3.2 displays the results: highly unionized region in dark-blue and lowly unionized region in light-blue.

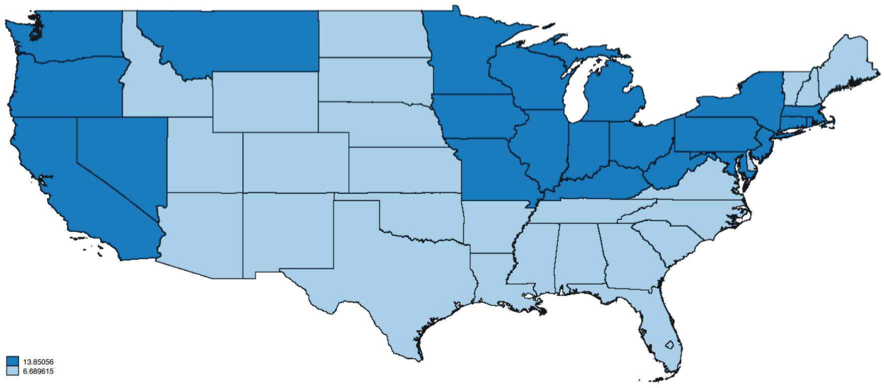
The stability of the cross-sectional standard deviation of the U.S. states unionization coverage over time suggests that the cross-sectional long-run unionization coverage *rankings* was preserved over time (Figure 1.B.3, Appendix 1.B.2).

Firm bankruptcy choices, by unionization region.

How did the shift in creditor rights affect the bankruptcy phenomenon of firms facing different bargaining power of workers? I answer this question in two steps.

First, I restrict the attention to the population of publicly listed firms in bankruptcy reorganization (UCLA LoPucki Bankruptcy Research Database).

Figure 2.3.2: Time series average coverage rate by unionization region over the period 1983-2014



Note: U.S. States organized in highly unionized (13.9%, dark blue) and lowly unionized (6.7%, light blue) region. *Source:* Union Membership and Coverage database (CPS), 1983-2014.

Table 2.3.1: Share of Successful Reorganizations Ch 11

Share of Successful Reorganizations Ch 11	**1979-1998**	**1999-2012**	**1979-2012**
Aggregate	0.9511	0.9201***	0.9296
Highly Unionized region	0.9661	0.9191 **	0.9310
Lowly Unionized region	0.9440	0.9208 **	0.9288

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$, one-sided mean-comparison t test (Welch)

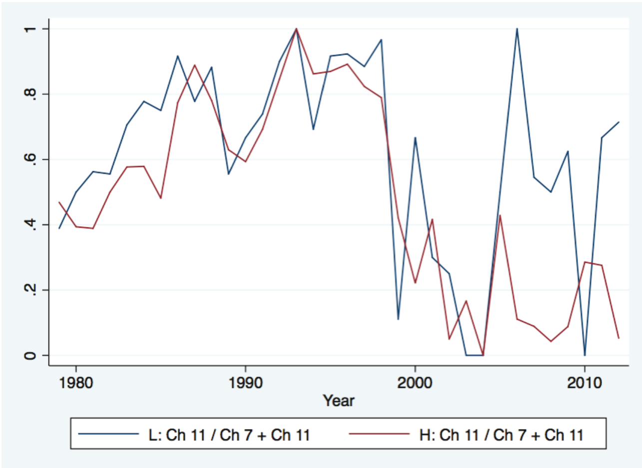
Note: The two-way table reports the ratio of the Ch 11 filings that are not converted to Ch 7 over the total number of Ch 11 filings for different regions (rows) and time periods (columns). Regions: the aggregate economy (row 1), highly unionized region (row 2), lowly unionized region (row 3); Time periods: Ch 11 cases that have been disposed in the pre-shift period (Column 1), post-shift period (Column 2) and the whole analysis period (Column 3). *Source*: UCLA LoPucki Bankruptcy Research Database, 1980-2012. The data-set is purged by involuntary filings, prepackaged cases, dismissals, and missing data.

The *restructuring channel* predicts that an increase in creditor rights reduces the incentives of shareholders to restructure labour contracts and makes more likely that reorganizations fail (*intensive margin channel*). In addition, the likelihood of success of Ch 11 should drop more as the bargaining power of worker increase, and a successful restructuring of labour contracts is required for the reorganization to succeed. To test these implications, I report the fraction of Ch 11 cases that are not converted to Ch 7 before and after the break, in aggregate and by unionization region (Table 2.3.1). In line with the legal literature, I document an economy-wide decrease in the likelihood of success of the Ch 11 procedure. I then complement the existing evidence, by documenting a larger drop in the likelihood of success of Ch 11 in the highly unionized region. The theory I will develop later is in line with this empirical evidence.

Second, I restrict the attention to the population of publicly listed firms in bankruptcy reorganization and liquidations (Compustat).

The *restructuring channel* predicts that an increase in creditor rights makes Ch 11 less attractive than Ch 7 (*extensive margin channel*), especially when workers extract a lot of rents, and restructuring labour contracts is key to regain economic soundness. In line with the theory, Figure 2.3.2 shows that after the break the relative use of Ch 11 drops significantly more for firms in the highly unionized region (red line).

Figure 2.3.3: Fraction of Ch 11 cases over total default, by unionization region



Note: Ratio of Ch 11 cases over total default (Ch 7 + Ch 11), in the lowly unionized region (blue), and highly unionized region (red). *Source:* Compustat North-America Fundamentals Annual, 1979-2012. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

Firm balance-sheets, by unionization region.

Table 2.3.2 takes a snapshot of the firms' distribution by unionization region, in aggregate, and for the time-windows of interest: pre-shift, post-shift and 1979-2012.
Few facts emerge. The median leverage decreased by 27% in the U.S., led by firms in highly unionized states (-43%). If any, leverage slightly increases in the lowly unionized area (1.3%). Similar behaviour for the dividend price ratio, that experienced a 46% drop at the country level, with firms in the highly unionized region on the driving seat (51% drop). While the median Tobin-q (market value of the firm over asset value) increases by almost 19% at any geographical level, the volatility more than quadruple at the country level, driven by firms in the lowly unionized region. In conclusion, labour productivity increased by almost 50% (median), and doubled its dispersion, without significant regional differences.

2.3.3 Regression Analysis

This section attempts to isolate the effect of the shift in creditor rights, by controlling for endogeneity issues that impairs the previous descriptive statistics analysis. Because of the

Table 2.3.2: Firm balance-sheets, by unionization region

	1979-1998			1999-2012			1979-2012		
	Mean	Median	Std	Mean	Median	Std	Mean	Median	Std
Leverage	0.1853	0.1360	0.1859	0.1865	0.0985	0.2321	0.1873	0.1247	0.2030
Lowly Unionized	0.2054	0.1635	0.1939	0.2263	0.1657	0.2410	0.2152	0.1678	0.2118
Highly Unionized	0.1745	0.1232	0.1799	0.1665	0.0702	0.2224	0.1733	0.1065	0.1955
Dividend Price Ratio	0.0082	0.0013	0.0140	0.0044	0.0000	0.0120	0.0061	0.0006	0.0129
Lowly Unionized	0.0078	0.0010	0.0143	0.0054	0.0000	0.0143	0.0066	0.0005	0.0146
Highly Unionized	0.0086	0.0016	0.0141	0.0042	0.0000	0.0111	0.0061	0.0007	0.0124
Tobin-Q	1.6140	0.9033	2.1469	2.8728	1.0786	8.5315	2.7442	1.0667	7.7504
Lowly Unionized	1.3677	0.7849	1.8287	2.6398	0.9351	8.3024	2.5188	0.9191	7.6358
Highly Unionized	1.7408	0.9789	2.2659	2.5643	1.1563	5.8050	2.4871	1.1453	5.3403
Labour Productivity	1.9884	1.4255	2.1400	3.2138	2.0986	4.2002	2.4245	1.6811	2.8542
Lowly Unionized	2.1660	1.3977	2.5805	3.8110	2.1136	5.3781	2.7132	1.6625	3.5065
Highly Unionized	1.9016	1.4427	1.8717	2.9852	2.1433	3.5005	2.2898	1.7043	2.4450

Note: The table reports values of the panel average, median and standard-deviation of the statistic reported in the row at different geographical level (Aggregate, Lowly Unionized, and Highly Unionized Region) for the pre-shift period (1979-1998), post-shift period (1999-2012), and the whole period (1979-2012). *Source*: Compustat North-America Fundamentals Annual, 1979-2012. Union Membership and Coverage database (CPS). The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR). It also excludes observations with missing State field.

absence of a control group, these findings do not have a causal interpretation and should be interpreted as suggestive.

The analysis is arranged in two layers. To motivate the question, I use *state*-level data and contrast the effect of the shift in creditor rights protection regime on labour productivity in highly and lowly unionized states. To substantiate the mechanism against competing hypothesis, I turn to *firm*-level data and investigate the impact of the shift on firms' bankruptcy outcomes, bankruptcy choices, leverage, and labour productivity.
Table 2.3.3 organizes the results accordingly. The analysis uses annual data over the period 1983-2012 from Compustat North-America Fundamentals Annual and the Union Membership and Coverage database (CPS)[24].

State-level Analysis

Table 2.3.3 Column 1 reports estimates of the following diff-in-diff regression

$$\ln \frac{Y_{t,s}}{L_{t,s}} = \alpha + \beta_U \cdot d_U + \beta_{>1998} \cdot d_{>1998} + \beta_{>1998,U} \cdot d_{>1998} \cdot d_U + \ln \frac{Y_{t-1,s}}{L_{t-1,s}} + \sigma' X_{t,s} + \alpha_t d_t + \alpha_s d_s$$

[24] The interested reader can refer to Appendix 1.A for a detailed description of the data.

Table 2.3.3: Main Results

	State Level	Firms Level			
	Labour Productivity	Bankruptcy Choice		Leverage	Labour Productivity
		Likelihood Success	Ch 11/Ch 7		
$d_{>1998}$	0.141***	1.516	8.306**	0.092***	0.022**
	(0.038)	(0.924)	(3.926)	(0.009)	(0.010)
$d_{>1998} \cdot d_U$	-0.099**	-5.169***	-3.005**	-0.026**	-0.035**
	(0.047)	(0.960)	(1.532)	(0.011)	(0.013)
Trend	No	No	No	Yes	Yes
Time dummies	Yes	No	Yes	No	No
State dummies	Yes	No	Yes	Yes	Yes
Sector dummies	No	No	Yes	Yes	Yes
Firms Controls	No	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes
Observations	1,477	300	177,988	148,949	140,847
$\hat{\beta}_{>1998} + \hat{\beta}_{>1998,U}$	0.042	-3.653	5.301	0.066	-0.013
s.e.	0.05	0.25	0.21	0.01	0.01

Regression coefficients, Standard error clustered at state level in parenthesis
$^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

Note: Regressions are organized in two panels: State Level and Firms Level. **State Level**: **Column (1) - Aggregate Labour Productivity**: Blundell-Bond two-steps estimates of the log of state level labour productivity, $\ln Y_{t,s}/N_{t,s}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit), number of firms, $N_{t,s}$. Time and states fixed effects are reported. *Instruments:* 1) GMM type: up to 3 lags of the dependent variable and continuous covariates; 2) iv-type: d_U, $d_{>1998}$, and $d_{>1998} \cdot d_U$. **Firms Level**: **Column (2) - Likelihood of Success of Ch 11**: Probit of Ch 11 (=1) vs. Chapter 7 (=0) over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *Firms level controls*: log real total assets, $a_{t,i}/P_t$, leverage, $b_{t,i}/a_{t,i}$. **Column (3) - Bankruptcy Choice**: Multinomial logit regressions of firms continuation choice $\phi = \{\text{Continuation}, \text{Ch 7}, \text{Ch 11}\}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *Country level controls*: level of union coverage, Cov_t; 3) *State level controls*: aggregate real sales, $Y_{t,s}$, Herfindhal index of sectoral concentration (sector real sales shares unit), employment shares by sector (naics), level of union coverage, $Cov_{t,s}$, interaction terms: $d_U \cdot Cov_{t,s}$, $d_{>1998} \cdot Cov_{t,s}$, $d_{>1998} \cdot d_U \cdot Cov_{t,s}$; 4) *Firms level controls*: real sales, $y_{t,i}/P_t$, leverage, $b_{t-1,i}/a_{t-1,i}$, real total assets, $a_{t,i}/P_t$; 5) *Fixed effect controls*: time, states, and sector fixed effects are reported. Ch 7 and Ch 11 denote the relative bankruptcy choices (the baseline case Ch 7 is omitted). **Column (4) - Leverage**: Fixed effect regression of $\ln b_{t,i}/a_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: lagged value, $\ln b_{t-1,i}/a_{t-1,i}$, log labour productivity, $\ln y_{t,i}/n_{t,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. Leverage is measured as total liabilities over total assets (compustat identifiers: lt, at). **Column (5) - Labour Productivity**: Fixed effect regression of $\ln y_{t,i}/n_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: log labour productivity, $\ln y_{t-1,i}/n_{t-1,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. *Source:* Compustat North-America Fundamentals Annual, 1979-2012. Union Membership and Coverage database (CPS), 1983-2014.

of log measured labour productivity $\ln \frac{Y_{t,s}}{L_{t,s}}$ over its lagged value, dummy variables $d_{(\cdot)}$ and a set of controls X, where the t and s subscripts stand for time and state. The identification assumption is that $\ln \frac{Y_{t,s}}{L_{t,s}}$ follows a stationary[25] AR(1) process. The regressors of interest are: the treatment, d_{1998}, which takes value 1 after the break in 1998; d_U, which takes value 1 when a state is highly unionized; the interaction term $d_{>1998} \cdot d_U$, which informs about the differential impact of the shift in creditor rights in highly unionized states. In addition to time, α_t, and state, α_s, fixed effects, the regression includes time-varying state-specific controls $X_{t,s}$: Herfindhal index of sectoral concentration (sector real sales shares unit), number of firms $N_{t,s}$.

After the break, log productivity increases by 14% in lowly unionized states, while it increases by a not significant $\beta_{>1998} + \beta_{>1998,U} = 2.6\%$ in highly unionized states. The estimates of $\beta_{>1998}$ and $\beta_{>1998,U}$ are stable across different methodologies (LSDV, Fixed Effect, Random Effect, Blundell-Bond) - which controls for a variety of sources of endogeneity and non-stationarity issues - suggesting a very precise estimation. The interested reader can refer to Appendix 1.B.3 for a detailed description of the empirical methodology adopted and robustness checks.

Firm-level Analysis

The order of the columns in Table 2.3.3 mirrors the unfolding of the mechanism. An increase in creditor rights reduces the incentive of shareholders to bargain with workers over the employment benefits, lowering the likelihood of success of Ch 11 in the highly unionized region - *intensive margin channel* (Table 2.3.1). As a result, the bankruptcy reorganization becomes less attractive than the liquidation alternative, especially in the highly unionized region - *extensive margin channel.* These channels imply lower recovery values upon default, and therefore more expensive debt, causing a shift towards a less leveraged capital structure, especially in highly unionized states - *general equilibrium channel.* The increase in the cost of debt per unit of collateral, makes more difficult to exploit investment opportunities reducing measured labour productivity, especially in the highly unionized region. Appendix 1.B.4 performs a battery of robustness and placebo checks.

2.4 The Static Model

The economy lasts for one period, and is populated by a representative firm, a representative worker and a mass one of identical lenders. The firm is run by a risk neutral shareholder and has access to a leontief technology that transforms capital, k, and a labour unit in output, $A \cdot \max\{k, \bar{k}\}$. The project scale, $\bar{k}$, denotes the threshold beyond which returns

[25] In Appendix 1.B.3 I perform a battery of panel data unit root tests followed by a state-by-state unit-root tests to support the assumption.

on capital sharply decrease. After production, the capital fully depreciates. The representative worker owns the firm, supplies a unit of labour inellastically, and consumes his income, $C = w + \Pi$, which consists of a wage, w, and firm's profits, Π. In conclusion, there is a competitive market of risk neutral lenders, which own $\bar{k}$[26] unit of capital that can lend to the representative firm or use to produce with a linear technology, k. To make the problem interesting, I assume that the firm's technology is more productive than the lenders' technology, $A > 1$.

The timing of the model is the following. At the beginning of the period the firm is paired with the worker, and lending takes place. At the end of the period the output is shared among the agents as follows. The shareholder chooses how much to borrow, k, to maximize profits

$$A \cdot \max\{k, \bar{k}\} - R_k \cdot k - w$$

The interest rate on the loan, R_k, and the wage, w, determines the shares of output that go to lenders and to the worker, respectively. On one side, price competition in the credit market bids the loan interest rate down to the return on the lenders' linear technology, $R_k = 1$. On the other side, the surplus of the firm - net of the capital repayment, k - is split between the shareholder and the worker in a nash bargaining fashion,

$$w = \arg\max_{v} \left[\, (A-1) \cdot k - v \,\right]^{1-\theta_U} \cdot \left[v - \underline{w} \right]^{\theta_U}$$

where θ_U denotes the bargaining power of the worker and $\underline{w}$ his outside opportunity. Since the worker does not value leisure, $\underline{w} = 0$.

An equilibrium in this economy is the set of prices $\{w, R_k\}$ and allocations such that the firm maximizes profits and the bond market clears.

This environment is peculiar. Resources are in the wrong hands: lenders own capital but do not have access to the productive technology, and viceversa the firm. In this economy, the misallocation arises if frictions impede that capital moves from the lenders to the firm. To study it, I compare outcomes under two polar assumptions on the enforceablity of debt contracts: perfect and limited.

2.4.1 Perfect enforceability of debt contracts

If debt contracts are perfectly enforceable, in equilibrium all capital is invested in the firm technology, $k^* = \bar{k}$[27]. Henceforth, I refer to the output under perfect enforceability of debt

[26]The fact that the aggregate amount of capital coincides with the project potential is for technical reasons (simplifies the algebra). Footnote 27 explains how.

[27]By assuming that the aggregate capital coincides with the project scale, we have that in first best all capital is invested in the productive technology. Assuming it differently - as long as the aggregate capital is greater than the project scale - would complicate the algebra but would keep unchanged the economic intuition.

contracts

$$Y^{FB} = A \cdot \bar{k} \tag{2.4.1}$$

as the first best aggregate output, and use it to contrast outcomes under limited enforceability of debt contracts.

2.4.2 Limited enforceability of debt contracts

This section introduces a twist: after producing, the shareholder can default on his debt obligations by filing either for reorganization (Ch 11) or liquidation (Ch 7). Bankruptcy is costly: upon bankruptcy all output is swiped out. In this environment, the firm value depends on the project scale. The two procedures differ on how they dispose of the project and share the surplus among the stakeholders: the shareholder, the lenders and the worker. Under Ch 7, lenders liquidate the project, net of a clearance loss $\psi \in (0, 1)$[28], and recover $(1 - \psi) \cdot \bar{k}$; the shareholder and the worker get nothing. Under Ch 11, the project is not liquidated and it is used to produce. Let $\zeta \cdot \bar{k}$[29] be the going-concern value of the firm in Ch 11, with $0 \leq (1 - \psi) < \zeta \leq 1 < A$. In Ch 11 all contracts $\{w, R_k\}$ are annulled[30]. As a result, the shareholder, lenders and the worker enter in a two stage nash bargaining process, which is structured as follows. The shareholder bargains first with the worker and afterwards with lenders. This assumption formalizes the higher priority that the U.S. Corporate bankruptcy law recognizes to the workers over lenders on the firm's surplus[31]. To conclude, the likelihood of success of the reorganization procedure, $\alpha^R(e, \theta_U)$, depends on the effort, $e \in [0, 1]$, that the shareholder decides to exert to restructure the labour contract, and the bargaining power of the worker, θ_U. In case of failure, the case is converted to Ch 7.

The shareholder solves the reorganization problem by backward induction. Accordingly, in the second stage he bargains with the lenders over a debt haircut, for a given wage compensation and level of effort (henceforth, the debt restructuring problem). Hence, in the first stage he bargains with the worker over the wage, for a given level of effort (henceforth, the labour restructuring problem). In conclusion, at the onset of the reorganization he chooses the level of effort that maximizes his expected share of the surplus, net of restructuring costs. The interested reader can refer to Appendix 1.C for all the derivations.

[28]The liquidation clearance loss captures the presence of frictions in the cash-auction procedure. As instance, the *financing problem* and *the lack of competition problem* (See Aghion et al. [1994]). The first problem relates to the difficulties in raising big amount of fundings in a brief amount time. The second problem depends on the lack of competition on the bidding sides. See Shleifer and Vishny [2011] for an amplification of the *financing problem* in recessions due to the congestion of the secondary markets (fire-sales).

[29]ζ captures the output distruption occurring during the reorganization: lost of supply-client relationship, best managers, etc..

[30]§365 (a) of the Bankruptcy Code.

[31]§507 (a) of the Bankruptcy Code.

2.4.3 The Debt Restructuring

In the second stage, for a given effort choice, e, and wage, v,

$$NB^C_{v,e}(\underline{k}) = \max_{r\in\mathbb{R}^+}\underbrace{[\alpha^R(e;\theta_U)\cdot(\zeta\cdot\bar{k}-v-r)]}_{\text{Firm's Expected Surplus}}{}^{1-\theta_C}\cdot\underbrace{[\alpha^R(e;\theta_U)\cdot r+(1-\alpha^R(e;\theta_U))\cdot(1-\psi)\cdot\bar{k}-(1-\psi)\cdot\bar{k}]}_{\text{Lenders' Expected Surplus}}{}^{\theta_C}$$

$$\text{s.t}\quad \zeta\cdot\bar{k}-v-r\geq 0\qquad \alpha^R(e;\theta_U)\cdot r+(1-\alpha^R(e;\theta_U))\cdot(1-\psi)\cdot\bar{k}\geq(1-\psi)\cdot\bar{k}\tag{2.4.2}$$

the shareholder and lenders bargain over the debt repayment, r, to maximize the nash bargaining product between their expected surpluses (the debt restructuring problem). In this context, the bargaining power of lenders, θ_C, proxies for the level of creditor rights protection. If the reorganization is unsuccessful the firm gets liquidated, the shareholder receives nothing, $V^L = 0$, and lenders get the recovery value under Ch 7, $(1-\psi)\cdot\bar{k}$. The specification of the surplus of the lenders formalizes a legal requirement referred to as the *best interest of creditors* test: by law it is responsibility of the judge to guarantee that creditors recover under Ch 11 at least as much as under Ch 7[32].

As a result of the bargaining process, the expected recovery value under Ch 11 and the expected surplus of the shareholder are, respectively,

$$R^{11}_{v,e}(\bar{k}) = (1-\psi)\cdot\bar{k}+\alpha^R(e;\theta_U)\cdot\theta_C\cdot\max\left[\zeta\cdot\bar{k}-(1-\psi)\cdot\bar{k}-v\ ,\ 0\right]\tag{2.4.3}$$

$$S^F_{v,e}(\bar{k}) = \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\max\left[\zeta\cdot\bar{k}-(1-\psi)\cdot\bar{k}-v\ ,\ 0\right].\tag{2.4.4}$$

2.4.4 The Labour Restructuring

In the first stage, for a given effort choice, e, the labour restructuring problem

$$NB^U_e(\underline{k}) = \max_{v\in\mathbb{R}^+}\ [\quad\underbrace{S^F_{v,e}(\bar{k})}_{\text{Firm's Expected Surplus}}\quad]^{1-\theta_U}\ \cdot\ [\alpha^R(e;\theta_U)\cdot v]^{\theta_U}\tag{2.4.5}$$

entails the choice of the wage compensation that maximizes the nash bargaining product between the expected surpluses of the shareholder and of the worker. Again, if Ch 11 fails the case is transferred to Ch 7, where both get nothing. Substituting (2.4.4) in (2.4.5) we get the wage compensation

$$w(\bar{k}) = \theta_U\cdot\max\left[\zeta-(1-\psi)\ ,\ 0\right]\cdot\bar{k}\tag{2.4.6}$$

Proposition 2.4.1. *The wage compensation upon labour restructuring decreases with the liquidation value* $(1-\psi)\bar{k}$. *(The threat of liquidation)*

[32]Pursuant to §1129 of Ch 11 impaired class of claims or interests 'will receive or retain under the plan on account of such claim or interest property of a value, as of the effective date of the plan, that is not less than the amount that such holder would so receive or retain if the debtor were liquidated under chapter 7 of this title on such date.

The shareholder uses the threat of liquidation in the second stage to reduce the bargaining position of the worker in the first stage. This conclusion results from the timing of the debt and labour restructuring problems, which arises from the order of priority in the payment assigned by the law to employees and creditors.
For the ease of notation, let

$$S(\bar{k}) \equiv \max\left[\zeta - (1-\psi)\ ,\ 0\right] \cdot \bar{k}$$

denote the nash bargaining surplus. Substituting (2.4.6) in the objective function, it is easy to show that - for a given effort level e - the expected recovery value under Ch 11 and the expected surpluses of the shareholder and worker are, respectively,

$$R_e^{11}(\bar{k}) = (1-\psi) \cdot \bar{k} + \alpha^R(e;\theta_U) \cdot \theta_C \cdot (1-\theta_U) \cdot S(\bar{k}) \tag{2.4.7}$$

$$S_e^F(\bar{k}) = \alpha^R(e;\theta_U) \cdot (1-\theta_C) \cdot (1-\theta_U) \cdot S(\bar{k}) \tag{2.4.8}$$

$$S_e^W(\bar{k}) = \alpha^R(e;\theta_U) \cdot \theta_U \cdot S(\bar{k}). \tag{2.4.9}$$

2.4.5 The Restructuring Effort Problem

Upon entering reorganization, the shareholder chooses the restructuring effort, $e \in [0,1]$, that maximizes his claims on the expected surplus of the firm,

$$V^R(\bar{k}) = \max_{e \in [0,1]} S_e^F(\bar{k}) - c(e,\bar{k})$$

I assume $\alpha^R(e;\theta_U) = (1-(1-e)\cdot\theta_U)$. This specification formalizes the intuition that without a formal attempt to restructure labour contracts, $e = 0$, the probability of success of the reorganization procedure decreases with the bargaining power of the worker, $1-\theta_U$. By exerting restructuring effort, the shareholder can temper this negative effect and increase the likelihood of success $(1-\theta_U + e\cdot\theta_U)$. Lastly, by assuming that the effort cost function is linear on the firm surplus, $c(e) \equiv c_{11} \cdot \frac{e^2}{2} \cdot S(\bar{k})$, the problem reads

$$V^R(\bar{k}) = \max_{e \in [0,1]} \left[(1-(1-e)\cdot\theta_U) \cdot (1-\theta_C) \cdot (1-\theta_U) - c_{11} \cdot \frac{e^2}{2} \right] \cdot S(\bar{k})$$

Proposition 2.4.2. *The optimal level of effort*

$$e^* = (1-\theta_C) \cdot (1-\theta_U) \cdot \frac{\theta_U}{c_{11}} \tag{2.4.10}$$

decreases with the bargaining power of lenders, θ_C.

An increase in creditor rights, θ_C, reduces the fraction of the nash-bargaining surplus that goes to the shareholder, tempering his incentives to exert effort. As a consequence, at optimum the likelihood of success of Ch 11

$$\alpha^R(e^*;\theta_U) = (1-\theta_U)\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right) - \frac{\theta_U^2}{c_{11}}\cdot\theta_C\right] \tag{2.4.11}$$

decreases with the creditor rights protection. In turn, this result implies that at optimum an increase in creditor rights has a countervailing effect on Ch 11 recovery values,

$$R^{11}(\bar{k}) = (1-\psi)\cdot\bar{k} + \underbrace{\alpha^R(e^*;\theta_U)\cdot\theta_C}_{\text{Trade off}}\cdot(1-\theta_U)\cdot S(\bar{k}) \tag{2.4.12}$$

If on one side, upon a successful reorganization it increases the recovery value (by increasing the share θ_C of the total surplus of the firms that goes to the lenders), on the other side it reduces the likelihood of success of Ch 11. As a result, an increase in creditor rights ought not to increase expected recovery values, and pro-creditor bankruptcy reforms can backfire. The next section, closes the model and studies how these reforms affect the allocation of resources in the economy.

2.4.6 Characterization of the equilbrium

The participation constraint, requires the debt repayment $R_k\cdot k$ to be no larger than the expected recovery value $L(\bar{k}) = \max[\,R^{11}(\bar{k})\,,\,R^7(\bar{k})\,]$[33],

$$R_k\cdot k \leq L(\bar{k})$$

By price competition in the credit market, $R_k = 1$, and the monotonicity of the firm's preferences over k we get the equilibrium level of borrowing

$$k^* = \min\left\{(1-\psi) + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C - \frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right]\cdot\max[\,\zeta-(1-\psi)\,,\,0\,]\,,\,1\right\}\cdot\bar{k} \tag{2.4.13}$$

Let $k^*/\bar{k}$ denote the fraction of resources that is invested in the firm's technology. Then, let $m = (\bar{k}-k^*)/\bar{k}$ denote the fraction of resources that is misallocated and invested in the un-productive linear technology. Then, the equilibrium output

$$Y = [1\cdot\underbrace{(1-m)}_{\text{Fraction of }\bar{k}\text{ invested in productive technology}} + \frac{1}{A}\cdot\underbrace{m}_{\text{Fraction of }\bar{k}\text{ invested in unproductive technology}}]\cdot\underbrace{A\cdot\bar{k}}_{Y^{FB}} \tag{2.4.14}$$

[33]The recovery value under bankruptcy has to be greater equal than the recovery value under Ch 7, independently on the procedure.

is smaller than the first best output, Y^{FB}. In turn, the misallocation of resources

$$m = 1 - \left[(1-\psi) + (1-\theta_U)^2 \cdot \underbrace{\left[\left(1 + \frac{\theta_U^2}{c_{11}}\right) \cdot \theta_C - \frac{\theta_U^2}{c_{11}} \cdot \theta_C^2 \right]}_{\alpha^R(e^*;\theta_U)\,\cdot\,\theta_C} \cdot \max\left[\zeta - (1-\psi) \,,\, 0 \right] \right]$$

depends on the relative efficiency of the bankruptcy law (ζ, ψ) and - through the labour and debt restructuring activity - depends on the level of creditor rights protection and bargaining power of workers, (θ_U, θ_C).

2.4.7 Normative Analysis

What is the (optimal) output maximizing level of creditor rights? To answer this question I study the problem of a social planner which takes as given the bargaining power of the worker, and maximizes output by choosing the level of creditor rights protection, θ_C. This problem is equivalent to the one of minimizing the misallocation of resources

$$\max_{\theta_C \in [0,1]} Y(\theta_C; \theta_U) = (1-\theta_U)^2 \cdot \max\left[\zeta - (1-\psi) \,,\, 0 \right] \max_{\theta_C \in [0,1]} \left[\left(1 + \frac{\theta_U^2}{c_{11}}\right) \cdot \theta_C - \frac{\theta_U^2}{c_{11}} \cdot \theta_C^2 \right] \tag{2.4.15}$$

Proposition 2.4.3. *In an interior solution*[34]*, the optimal level of creditor rights*

$$\theta_C^*(\theta_U) = \frac{1}{2} \cdot \left[\frac{c_{11}}{\theta_U^2} + 1 \right] \tag{2.4.16}$$

decreases with the bargaining power of the worker.

The existence of a blissing point in the social planner problem is the result of the countervailing effect that an increase in creditor rights has on the equilibrium likelihood of success of Ch 11, $\alpha^R(e^*; \theta_U)$, and on the share of nash bargaining surplus that goes to the lenders, $(1-\psi) \cdot \bar{k} + \alpha^R(e^*; \theta_U) \cdot \theta_C \cdot (1-\theta_U) \cdot S(\bar{k})$. Keeping fix $\alpha^R(e^*; \theta_U)$, an increase in θ_C mechanically increases the share of the nash bargaining surplus, $S(\bar{k})$, that goes to the lenders. Upon a successful reorganization, it increases recovery values, fosters ex-ante the lending, and reduces the misallocation of resources in the economy, boosting output. Yet, it reduces the share of the nash bargaining surplus that goes to the shareholder, $(1 - \theta_C) \cdot (1-\theta_U) \cdot \alpha^R(e^*; \theta_U) \cdot S(\bar{k})$, tempers his incentives to exert restructuring effort, and, by doing so, reduces the likelihood of success of Ch 11, (2.4.11).

[34] When $\zeta > (1-\psi)$, or equivalently when, $R^{11}(\bar{k}) > R^7(\bar{k})$.

Proposition 2.4.3 says that whether one force prevails the other depends on the bargaining power of the worker. When $\theta_U \to 0$, exerting effort to restructure labour contracts, e, does not help the firm to regain economic soundness, and therefore does not alter the likelihood of success of Ch 11, $\alpha^R(e^*; \theta_U)$. It is then optimal to give all the bargaining power to the lenders, $\theta_C \to 1$. Conversely when $\theta_U \to 1$, failing in restructuring labour contracts can prevent the firm to regain its economic soundness, and therefore exerting effort in restructuring labour contracts is crucial. Accordingly, the optimal level of creditor rights attains its minimum, $0.5(c_{11} + 1)$.

Out of the algebra, Proposition 2.4.3 contains an important message. In economies where worker extract a lot of rents, an increase in creditor rights can increase the misallocation of resources, by suffocating the incentives to restructure labour contracts in reorganization. This result might shed light on why more unionized countries - as Italy, France - have lower creditor rights protection than less unionized ones - say, U.S.

2.5 The Dynamic Model

The economy is populated by firms, credit intermediaries and a household. In the economy there are two regions. Regions differ by the bargaining power of workers - highly and lowly unionized - and by the measure of firms[35].

Firms are run by risk neutral shareholders[36], who maximize the expected discounted stream of dividends. They articulate in two types: incumbents and entrants.

There is a continuum of incumbents, which differ by the region where they are located, their (fixed) capital scale of production, their (fixed) productivity and their histories. The incumbents are the producing firms in the economy. They combine capital and labour into a decreasing returns-to-scale technology and experience uninsurable persistent idiosyncratic productivity shocks. The labour cost varies across regions and across firms: the (region-specific) bargaining power of workers determine the fraction of the (firm-specific) surplus that is extracted by workers. Incumbents finance investment and dividends using internal and external funds: retained profits, one-period non-contingent loans and equity issuance. Incumbents can renege on their debt obligations and default. In compliance with the bankruptcy law, they have access to two bankruptcy procedures: liquidation (Ch 7) and reorganization (Ch 11). In Ch 7 an incumbent relinquishes all its assets to the creditors (net of a liquidation loss) and exits from the market; workers are laid off. Conversely, in Ch 11 an incumbent enters in a reorganization procedure with the other stakeholders: workers and creditors. The process articulates in a two stage [nash]

[35]The household and the credit intermediaries abstract from the spatial dimension.

[36]The model abstracts from agency frictions arising from the separation of governance and control. Recent empirical studies suggests that managers vs shareholders does not characterize the key tension in large corporate reorganizations, where 70% of CEO are replaced within 2 years of the bankruptcy filing (Ayotte and Morrison [2009]).

bargaining with workers (first) and creditors (later) over labour and debt contracts. The success of the Ch 11 procedure is stochastic. If the reorganization fails, the case is transferred to Ch 7. By exerting costly effort in restructuring labour contracts, the firm can increase the likelihood of success of the procedure.

In each period, a positive mass of potential entrants starts production with a time-to-build lag. At entry, each firm draws a region, a capital-scale, and a permanent productivity level, that remain fixed over life. Then it draws a persistent idiosyncratic productivity shock and decide whether to *actually* enter or not. Actual entrants finance their capital scale by issuing equity or debt.

Firms have access to a competitive financial sector with free entry. Each financial intermediary offers a menu of loan sizes and interest rates to firms, wherein each loan makes zero expected profits.

In conclusion, the representative household owns the firms, saves in the credit market, supplies inellastically labour to firms, and consumes out of the wage income, returns on savings and the aggregate amount of dividends distributed by firms.

2.5.1 The State of the Incumbents

An incumbent is defined as a tuple (r, k, z, b, x) where: $r \in \mathrm{R} \equiv \{L, H\}$ is the index of the location, where L [H] denotes the lowly [highly] unionized region; $k \in \mathrm{K} \equiv [k_{min}, k_{max}] \subset \mathbb{R}_{+}$ is the physical capital stock scale, as drawn at entry; $z \in \mathrm{Z} \equiv [z_{\min}, z_{\max}]$ denotes the permanent productivity, as drawn at entry; $x \in \mathrm{X} \equiv [x_{\min}, x_{\max}] \subset \mathbb{R}_{+}$ is an uninsurable idiosyncratic productivity shock; $b \in \mathrm{B} \equiv \{b_{\min}, \ldots, b_{\max}\} \subset \mathbb{R}$ is the amount of outstanding debt/savings, where B is a finite set with cardinality $|\mathrm{B}|$, and $b_{\min} < 0$, $b_{\max} > 0$.

To simplify notation, I summarize with $\underline{p} \equiv (r, k, z) \in \underline{P} \equiv \mathrm{R} \times \mathrm{K} \times \mathrm{Z}$ the permanent characteristics, and with $\underline{s} \equiv (b, x) \in \underline{S} \equiv \mathrm{B} \times \mathrm{X}$ the endogenous state variables.

2.5.2 The Production Technology

Firms use capital, k, and labour, $n \in \mathrm{N} \equiv [n_{min}, n_{max}] \subset \mathbb{R}_{+}$, to produce an homogeneous consumption good, $y \in \mathrm{Y} \subset \mathbb{R}_{+}$, using a decreasing returns-to-scale production technology,

$$y(\underline{p}, x, n) \equiv (z \cdot x)^{(1-\alpha\eta)} (k^{1-\alpha} n^{\alpha})^{\eta} \tag{2.5.17}$$

where[37] η is the decreasing return to scale parameter[38] and α is the value-added share of labour. The idiosyncratic productivity, x, follows a stochastic process defined on the mea-

[37] The normalization parameter $(\cdot)^{1-\alpha\eta}$ on the actual productivity level, $z \cdot x$, ensures that the firm's profit function after wage compensation $\pi(\underline{p}, \underline{s})$ is linear on $z \cdot x$

[38] The parameter $(1 - \eta)$ is sometimes referred to as the *span of control* (Lucas [1978]).

surable space $(X, \mathcal{B}(\mathrm{X}))$ with transition function $Q(x, \mathrm{d}x')$[39], where hereafter $\mathcal{B}(\cdot)$ denotes the Borel algebra on X. The operating profits are

$$\pi(\underline{p}, \underline{s}, n) \;\equiv\; y(\underline{p}, x, n) - w(\underline{p}, \underline{s}, n) \cdot n - \chi_o \tag{2.5.18}$$

where the wage contracts $(\underline{p}, \underline{s}, n, w(\underline{p}, \underline{s}, n)) \in W(\underline{p}, \underline{s}, n)$ are firm specific and determined through nash bargaining, as described in Section 2.5.7. Capital depreciates at rate δ. Thereby, to maintain a constant capital scale, investment equals $i = \delta k$[40].

2.5.3 The Financing Technology

Incumbents finance investment using retained profits, one-period non-contingent loans and equity issuance[41]. Let

$$g(d) \equiv \left[\mathbb{I}_{\{d \geq 0\}} + \iota \cdot \mathbb{I}_{\{d<0\}}\right] \cdot d$$

denote the flow of dividends [equity issuance] $d \in \mathrm{D} \equiv [\underline{d}, \bar{d}] \subset \mathbb{R}$ between the household and the firm. Henceforth, as a convention, let $\mathbb{I}_{\{y\}}$ denote an indicator function, which takes value 1 when y is true. The previous formula says that firms can issue equity by setting $d < 0$ and incurring an additional proportional cost, $\iota > 1$[42].

The presence of a default option yields a substantial departure of the loan-market arrangement from the Arrow-Debreu world. This departure formalizes in the device of *firm specific* one-period non-contingent loan contracts, $\left(\underline{p}, x, b', q(\underline{p}, x, b')\right)$, where $q : \underline{P} \times \underline{S} \rightarrow \mathrm{Q}$ is the pricing function in the space of continuous and bounded functions $\mathscr{C}^{\mathrm{Q}}$, with $\mathrm{Q} \equiv [0, q_{\max}] \subseteq \mathbb{R}, 0 \leq q_{\max} \leq 1$. In particular, a firm with characteristics $(\underline{p}, x)$ is allowed to save $(b' < 0)$ or borrow $(b' > 0)$ at the price $q(\underline{p}, x, b')$. This specification highlights the dependence of the loan price on five key firms characteristics: the permanent (z) and persistent productivity (x), the assets (k), the region (r), and the size of the loan (b'). If shocks are persistent[43], a high productivity today, x, predicts a higher productivity next period. Thereby, the firm is less likely to default and can issue debt at a higher price. Similar argument holds for z. A higher capital scale, k, yields a larger collateral, which tempers creditors' losses upon default, and mitigates downward pressures on the debt price. The region, r, affects the labour cost and therefore the firm's profitability; by doing

[39] I assume that $Q(x, \mathrm{d}x')$ is continuous on (x, x'), is decreasing in a first order stochastic dominance sense on x. This property is satisfied by many processes - e.g. the first order autoregressive process on which I focus in the calibration - and capture the idea that the higher is the idiosyncratic productivity today, the more likely it will be higher tomorrow. On the top of that, I assume that $Q(x, \mathrm{d}x')$ satisfies the Feller property.

[40] The result follows from the law of motion of capital $k' = (1 - \delta)k + i$ and the fact that $k' = k$.

[41] To maintain tractable the state space, I do not consider the outright hierarchical layers of ownership (bonds, debentures, preferred equity, common equity) but just a neat pattern of layered debt and equity.

[42] Following the literature, equity issuance is expensive (Hennessy and Whited [2007]).

[43] If productivity shocks were i.i.d., the price q would not depend anymore on the current x.

so, it alters the likelihood of financial distress and the associated interest rate on debt. In conclusion, larger loans increase the probability of default and commands higher interest rates.

As a result, firms preferences over the financing sources are in line with the pecking order theory: first retained profits, then debt and only then equity issuance. In *equilibrium* , the equity issuance cost, $\iota - 1$, establishes a lower bound to the debt price.

2.5.4 The Firm Choices

Firms take ordinary decisions (n, d, b'), and extra-ordinary decisions. In particular, they decide whether to continue ($\phi_X = 0$) or exit ($\phi_X = 1$). Upon exit, they decide whether to default ($\phi_D = 1$) or repay the debt ($\phi_D = 0$). If they default, they have to choose the bankruptcy procedure: reorganization ($\phi_R = 1$) or liquidation ($\phi_R = 0$).

2.5.5 The Bankruptcy Law Technology

The bankruptcy procedures formalize the legal ways by which firms can repudiate their debt obligations. In the model, a *bankruptcy procedure*, ϕ_R, is: 1) a set of stipulations, $S^R_{\phi_R} \in \mathbb{R}^3$, on firms' ordinary decisions, (n, d, b'); 2) a legal environment whereby stakeholders[44] agree on how to split the surplus; 3) a resolution about the existence of the firm as a going concern.

Bankruptcy Liquidation

In bankruptcy liquidation ($\phi_R = 0$), a firm does not produce, and cannot take any ordinary decision (n, d, b')[45], $S^R_0(\underline{p}, \underline{s}) \equiv \varnothing$. Creditors seize the collateral of the firm - which consists of undepreciated capital -

$$R^7(\underline{p}, \underline{s}) \equiv \min\{\ b\ ,\ (1-\psi)(1-\delta)k\ \} \tag{2.5.19}$$

suffering a liquidation clearance loss, $\psi \in (0, 1)$, which captures frictions in the cash-auction procedure[46]. Workers and shareholders get nothing. Once the firm is liquidated, it exits the market.

[44] The definition of stakeholders include: shareholders, workers and credit intermediaries.

[45] Indeed, in Ch 7 the firm ceases the ordinary activity. The judge appoints a trustee with the precise purpose of marshalling the assets of the firm and reimburse the creditors.

[46] As instance, the *financing problem* and *the lack of competition problem* (See Aghion et al. [1994]). The first problem refer to the difficulties of raising large fundings in short time. The second problem arise from the lack of competition on the bidding sides. See Shleifer and Vishny [2011] for a study of the amplification of the *financing problem* in recessions due to the congestion of the secondary markets (fire-sales).

Bankruptcy Reorganization

During bankruptcy reorganization, a firm cannot distribute dividends, and cannot save[47][48]

$$
\begin{aligned}
&S_1^R(\underline{p}, \underline{s}) \equiv \Big\{(n, d, b') \in \mathrm{N} \times \mathrm{D}_{-} \times B_{+} : \\
&d - q(\underline{p}, x, b')b' + k \leq y(\underline{p}, x, n) - w^R(\underline{p}, \underline{s}, n) \cdot n - \chi_o + (1-\delta)k - \alpha^C(\underline{p}, \underline{s})b\Big\}
\end{aligned}
\tag{2.5.20}
$$

The reorganization procedure involves a two stage nash bargaining, first with workers over labour contracts, $w^R(\underline{p}, \underline{s})$, and then with creditors over debt contracts, $\alpha^C(\underline{p}, \underline{s})b$. Reorganizations can fail. By exerting costly effort (in restructuring labor contracts) the firm can increase the likelihood of success. Section 2.5.8 elaborates the details.

2.5.6 The Timing

The timing is the following: i) productivity shocks realize and incumbents decide ii) whether to continue, to exit or to default; iii.a) if they continue, they produce and take dividend, investment and financing decisions; iii.b) if they exit, they sell the assets, and use the proceedings to repay the debt and distribute dividends (if any); iii.c) if they file for liquidation, they do not produce and exit the market; iii.d) if they file for reorganization, they produce, restructure labour expenses, bargain over a debt haircut and take financing decisions (*jointly* with the creditors); if the reorganization succeeds they continue, otherwise they are liquidated.

The Incumbents

Let the value $V : \underline{P} \times \underline{S} \to \mathbb{R}$ of an incumbent $(\underline{p}, \underline{s})$ be

$$
V(\underline{p}, \underline{s}) = \max_{\phi_X} \left\{ \underbrace{V^C(\underline{p}, \underline{s})}_{\text{Continuation}} , \max_{\phi_R} \left\{ \underbrace{V^X(\underline{p}, \underline{s})}_{\text{Exit}} , \max_{\phi_D} \left\{ \underbrace{V^R(\underline{p}, \underline{s})}_{\text{Reorganization}} , \underbrace{V^L(\underline{p}, \underline{s})}_{\text{Liquidation}} \right\} \right\} \right\}
\tag{2.5.21}
$$

where

$$V^X(\underline{p}, \underline{s}) = (1-\delta)k - b$$

denotes the value at exit,

$$V^L(\underline{p}, \underline{s}) = 0$$

[47] These restrictions arise from an application of the Absolute Priority Rule: to secure the higher priority of creditors' claims over the shareholders' ones, most bankruptcy laws do not allow firms to divert funds (by distributing dividends or save).

[48] $Y_{-} = Y \setminus \mathbb{R}_{++}$, and $Y_{++} = Y \setminus \mathbb{R}_{-}$.

denotes the value of liquidation, and $V^C(\underline{p}, \underline{s})$, $V^R(\underline{p}, \underline{s})$ denote the value of a firm that decides, respectively, to continue and reorganize, as characterized in the following sections.

2.5.7 The Continuation Problem

Let

$$
\begin{aligned}
V^C(\underline{p}, \underline{s}) = & \max_{(n,d,b')\in \mathrm{N}\times\mathrm{D}\times\mathrm{B}} g(d) + \beta \cdot \mathbb{E}_{x'|x}\left[V(\underline{p}, \underline{s}')\right] \\
& \text{s.t.} \quad d \leq y(\underline{p}, x, n) - w(\underline{p}, \underline{s}, n) \cdot n - \chi_o - \delta k + q(\underline{p}, x, b')b' - b \\
& \qquad (\underline{p}, \underline{s}, n, w(\underline{p}, \underline{s}, n)) \in W(\underline{p}, \underline{s}, n)
\end{aligned}
\tag{2.5.22}
$$

describe the problem of a firms that decides to continue. The menu of wage contracts $(\underline{p}, \underline{s}, n, w(\underline{p}, \underline{s}, n)) \in W(\underline{p}, \underline{s}, n)$ defines the wage compensation $w(\underline{p}, \underline{s}, n)$ to be paid by a firm $(\underline{p}, \underline{s})$ that hires n workers. Let $\theta_U(r) \in [0, 1]$ denote the bargaining power of workers, which varies across regions, with $\theta_U(L) < \theta_U(H)$. Let $w : \underline{P} \times \underline{S} \times \mathrm{N} \to \mathrm{W} \equiv [0, w_{\max}]$ be the wage function in the space $\mathscr{C}^{\mathrm{W}}(\underline{P} \times \underline{S} \times \mathrm{N})$ of continuous functions bounded between $[0, w_{\max}]$.

Then, I can define the wage correspondence $(Ww) : \mathrm{W} \subseteq \mathbb{R}^+ \to \mathbb{R}^+$ as

$$
\begin{aligned}
(Ww)(\underline{p}, \underline{s}, n) \equiv & \arg\max_{v\in \mathrm{W}} \; V^C_{v,n}(\underline{p}, \underline{s})^{(1-\theta_U(r))} \cdot [v \cdot n - \underline{w} \cdot n]^{\theta_U(r)} \\
\text{s.t. } & V^C_{v,n}(\underline{p}, \underline{s}) \geq 0, \qquad v \geq \underline{w}
\end{aligned}
\tag{2.5.23}
$$

where $\underline{w}$ is the wage that satisfies the free entry condition, as specified in Section 2.5.9 and

$$
V^C_{v,n}(\underline{p}, \underline{s}) \equiv \max_{b'\in \mathrm{B}} g\left[y(\underline{p}, x, n) - \delta k - \chi_o + q(\underline{p}, x, b') \cdot b' - b - v \cdot n\right] + \beta \mathbb{E}_{x'|x}\left[V(\underline{p}, \underline{s}')\right],
$$

is the continuation value when the wage is v and the number of workers hired is n.

Theorem 2.5.1. *There exists a unique $w^* \in \mathscr{C}^{\mathrm{W}}(\underline{P} \times \underline{S} \times \mathrm{N})$ such that $w^* = (Ww^*)$.*

Proof. See Appendix 1.D.1. □

Proposition 2.5.2. *Given a number of workers, n, and a continuing firm $(\underline{p}, \underline{s})$, in an interior solution:*
- the nash bargaining surplus is

$$
S(\underline{p}, \underline{s}, n) \equiv \max_{b'\in \mathrm{B}} y(\underline{p}, x, n) - \delta k - \chi_o + q(\underline{p}, x, b') \cdot b' - b - \underline{w} n + \beta \cdot \frac{1}{\mathbb{I}_{\{d\geq 0\}} + \iota \cdot \mathbb{I}_{\{d<0\}}} \cdot \mathbb{E}_{x'|x}\left[V(\underline{p}, \underline{s}')\right]
\tag{2.5.24}
$$

- the wage compensation is

$$w(\underline{p}, \underline{s}, n) \cdot n = \underline{w} \cdot n + \theta_U(r) \cdot S(\underline{p}, \underline{s}, n) \tag{2.5.25}$$

Accordingly, the continuation problem (2.5.22) *can be rewritten as*

$$V^C(\underline{p}, \underline{s}) = (1 - \theta_U(r)) \max_{n \in \mathrm{N}} S(\underline{p}, \underline{s}, n) \tag{2.5.26}$$

Proof. See Appendix 1.D.2. □

Proposition (2.5.2) says that the continuation problem (2.5.22) boils down to (2.5.26): the firm chooses (n, b') to maximize the share of expected discounted value of future dividends that is not extracted by workers, (2.5.26). Appendix 1.D.2 discusses the separability of the max operator over the firm's choices, b' and n. Proposition 2.5.3 characterizes the problem.

Proposition 2.5.3. *The labour demand, n^*, and output, y^*, of firm $(\underline{p}, \underline{s})$ are*

$$n^*(\underline{p}, \underline{s}) = z \cdot x \cdot \left(\frac{\alpha\eta}{\underline{w}}\right)^{\frac{1}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \quad (2.5.27) \qquad y^*(\underline{p}, \underline{s}) = z \cdot x \cdot \left(\frac{\alpha\eta}{\underline{w}}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \quad (2.5.28)$$

Proof. See Appendix 1.D.2. □

Proposition 2.5.3 says that the firm's labour demand does not depend on $w(\underline{p}, \underline{s}, n)$, but on the outside opportunity cost, $\underline{w}$. In words, this result means that firm and workers' interests are aligned in making the pie as big as possible, $S(\underline{p}, \underline{s}, n^*)$, and contrast only on how to split it, $w(\underline{p}, \underline{s}, n^*)$. It also means, that the class of wage contracts studied is the *weakest*, in the sense that it does not *directly* distort ordinary decisions - say, to generate inefficient size choices - but it distorts *directly* only extensive margin ones: entry, exit and default.

Substituting the optimal choices, the wage compensation becomes

$$w(\underline{p}, \underline{s}) = \underline{w} + \theta_U \cdot \frac{S(\underline{p}, \underline{s}, n^*)}{n^*} \tag{2.5.29}$$

2.5.8 The Reorganization Problem

Let

$$\begin{aligned} V^R(\underline{p}, \underline{s}) = \max_{e \in \mathrm{E}} \alpha^R(e; \theta_U(r)) \cdot \left[\max_{(n,d,b') \in \mathrm{N} \times \mathrm{D}_- \times \mathrm{B}_+} g(d) + \beta \cdot \mathbb{E}_{x'|x} \left[V(\underline{p}, \underline{s}') \right] \right] - c(e) \\ \text{s.t.} \quad d \le y(\underline{p}, x, n) - \underbrace{w^R(\underline{p}, \underline{s}, e, n)}_{\text{Labour Restructuring}} \cdot n - \chi_o - \delta k + q(\underline{p}, x, b') \cdot b' - \underbrace{\alpha^C(\underline{p}, \underline{s}, e, n, w^R(\underline{p}, \underline{s}, e, n))}_{\text{Debt Restructuring}} \cdot b \end{aligned} \tag{2.5.30}$$

$$\begin{aligned} (\underline{p}, \underline{s}, e, n, v, \alpha^C(\underline{p}, \underline{s}, e, n, v)) \in A^C(\underline{p}, \underline{s}, e, n, v) \\ (\underline{p}, \underline{s}, e, n, w^R(\underline{p}, \underline{s}, e, n)) \in W^R(\underline{p}, \underline{s}, e, n) \end{aligned} \tag{2.5.31}$$

describe the problem of a firm that decides to file for Ch 11. In reorganization, shareholders enter in a two stage nash bargaining, first with workers over the wage compenation w^R (labour restructuring problem) and then with creditors over the debt haircut α^C (debt restructuring problem). The timing reflects the super-priority that the U.S. Corporate bankruptcy law recognizes to workers claims over creditors' ones on the firm's surplus. The reorganization succeds with probability $\alpha^R(e, \theta_U(r))$, that depends on the restructuring effort, e, and the regional bargaining power of workers, $\theta_U(r)$. By backward induction, shareholders solve, first, the debt restructuring problem (for a given effort choice, number of workers, and wage), and then the labour restructuring problem (for a given effort choice). In conclusion, upon entering reorganization, they choose the effort that maximizes their share of the expected discounted value of future dividends net of a restructuring cost. The next sessions develop the details.

The Debt Restructuring

In the second stage, for a given effort choice, e, number of workers, n, and wage, v, shareholders bargain with the credit intermediaries over the due recovery rate $a \in [0, 1]$ on the defaulted loan, b. Let the expected surplus of a firm $(\underline{p}, \underline{s})$ that files for reorganization be

$$S^F_{e,n,v}\left(\underline{p}, \underline{s}; a\right) \equiv \alpha^R(e; \theta_U(r)) \cdot \max\left\{ \max_{b' \in B_+} \iota \cdot \left[y(\underline{p}, x, n) - v \cdot n - \chi_o - \delta k + \underbrace{q(\underline{p}, x, b') \cdot b'}_{\text{D.I.P. Financing}} - ab \right] + \beta \cdot \mathbb{E}_{x'|x}\left[V(\underline{p}, \underline{s}') \right] , 0 \right\}$$

$$\text{s.t. } y(\underline{p}, x, n) - v \cdot n - \chi_o - \delta k + \underbrace{q(\underline{p}, x, b') \cdot b'}_{\text{D.I.P. Financing}} - ab \leq 0 \qquad (\textit{Equity Issuance}) \tag{2.5.32}$$

Let the credit intermediaries surplus be

$$S^C_{e,n,v}\left(\underline{p}, \underline{s}; a\right) \equiv \min\left\{ b\,,\ \underbrace{\max[\, \alpha^R(e; \theta_U(r)) \cdot a \cdot b + (1 - \alpha^R(e; \theta_U(r))) \cdot R^7(\underline{p}, \underline{s})\,,\ R^7(\underline{p}, \underline{s})\,]}_{\text{Best Interest of creditors test}} \right\} \tag{2.5.33}$$

The minimum operator controls for the fact that creditors cannot recover more than the outstanding debt. Conversely, the maximum operator formalizes the following legal requirements - sometimes referred to as the *best interest of creditors* test[49]: it is responsibility of the judge to guarantee that creditors recover under Ch 11 at least as much as under Ch 7.

Let $\theta_C \in (0, 1)$ denote the bargaining power of creditors and $\alpha^C : \underline{P} \times \underline{S} \times \mathrm{E} \times \mathrm{N} \times \mathrm{W} \rightarrow$

[49]Pursuant to §1129 of Ch 11 impaired class of claims or interests 'will receive or retain under the plan on account of such claim or interest property of a value, as of the effective date of the plan, that is not less than the amount that such holder would so receive or retain if the debtor were liquidated under chapter 7 of this title on such date.

$\mathrm{A} \equiv [0,1]$ denote the Ch 11 recovery rate function. Then, I can define the reorganization recovery rates correspondence $(A^C\alpha^C) : \mathrm{A} \to \mathbb{R}^+$ as

$$(A^C\alpha^C)(\underline{p}, \underline{s}, e, n, v) \equiv \arg\max_{a\in[0,1]} \left\{ \underbrace{[S^F_{e,n,v}\left(\underline{p}, \underline{s}; a\right)]}_{\text{Surplus Firm}}{}^{(1-\theta_C)} \cdot \underbrace{[\, S^C_{e,n,v}\left(\underline{p}, \underline{s}; a\right)\,]}_{\text{Surplus Creditors}}{}^{\theta_C} \right\} \tag{2.5.34}$$

$$\text{s.t. } S^F_{e,n,v}\left(\underline{p}, \underline{s}; a\right) \geq 0, \quad S^C_{e,n,v}\left(\underline{p}, \underline{s}; a\right) \geq 0$$

Theorem 2.5.4. *There exists a unique* $\alpha^{C,*} \in \mathscr{C}^{\mathrm{A}}(\underline{P} \times \underline{S} \times \mathrm{E} \times \mathrm{N} \times \mathrm{W})$ *such that* $\alpha^{C,*} = (N\alpha^{C,*})$.

Proof. See Appendix 1.D.1. □

Proposition 2.5.5. *Given* (e, n, v)*, and a reorganizing firm* $(\underline{p}, \underline{s})$*, in an interior solution:*
- the nash bargaining surplus is

$$S^R_{n,v}(\underline{p}, \underline{s}) \equiv \max \left\{ \max_{b'\in \mathrm{B}_+} y(\underline{p}, x, n) - \chi_o + q(\underline{p}, x, b')b' - \delta k - vn + \beta \cdot \frac{1}{\iota} \cdot \mathbb{E}_{x'|x}\left[V(\underline{p}, \underline{s}')) \right] - R^7(\underline{p}, \underline{s}), 0 \right\} \tag{2.5.35}$$

- the expected Ch 11 recovery value is

$$R^{11}_{e,n,v}(\underline{p}, \underline{s}) \equiv \alpha^C(\underline{p}, \underline{s}, e, n, v)b = R^7(\underline{p}, \underline{s}) + \alpha^R(e; \theta_U(r)) \cdot \theta_C \cdot S^R_{n,v}(\underline{p}, \underline{s}) \tag{2.5.36}$$

- the share of expected surplus that goes to shareholders is

$$S^F_{e,n,v}(\underline{p}, \underline{s}) \equiv \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot \iota \cdot S^R_{n,v}(\underline{p}, \underline{s}) \tag{2.5.37}$$

Proof. See Appendix 1.D.3. □

The Labour Restructuring

In the first stage, for a given effort choice, e, and number of workers, n, shareholders bargain with workers over the wage compensation. Let $w^R : \underline{P} \times \underline{S} \times \mathrm{E} \times \mathrm{N} \to \mathrm{W} \equiv [0, w_{\max}]$ be the wage function in the space $\mathscr{C}^{\mathrm{W}}(\underline{P} \times \underline{S} \times \mathrm{E} \times \mathrm{N})$ of continuous functions bounded between $[0, w_{\max}]$. Then, I can define the wage correspondence $(W^R w^R) : \mathrm{W} \subseteq \mathbb{R}^+ \to \mathbb{R}^+$ as

$$(W^R w^R)(\underline{p}, \underline{s}, e, n) \equiv \arg\max_{v\in \mathrm{W}} [S^F_{e,n,v}(\underline{p}, \underline{s})]^{(1-\theta_U(r))} \cdot [\alpha^R(e; \theta_U(r)) \cdot (v \cdot n - \underline{w} \cdot n)]^{\theta_U(r)} \tag{2.5.38}$$

$$\text{s.t. } S^F_{e,n,v}(\underline{p}, \underline{s}) \geq 0, \qquad v \geq \underline{w}$$

where I assume that workers get nothing if the reorganization procedure fails (and the case is transferred to Ch 7).

Theorem 2.5.6. *There exists a unique* $w^{R,*} \in \mathscr{C}^{\mathrm{W}}(\underline{P} \times \underline{S} \times \mathrm{E} \times \mathrm{N})$ *such that* $w^{R,*} = (W^R w^{R,*})$.

Proof. See Appendix 1.D.1. □

The menu of wage contracts $(\underline{p}, \underline{s}, n, e, w^R(\underline{p}, \underline{s}, e, n)) \in W(\underline{p}, \underline{s}, e, n)$, establishes the wage compensation $w^R(\underline{p}, \underline{s}, e, n)$ that a firm $(\underline{p}, \underline{s})$ has to pay when decides to hire n workers, and exert e effort in restructuring labour contract.

Proposition 2.5.7. *Given* (n, e)*, and a reorganizing firm* $(\underline{p}, \underline{s})$*, in an interior solution:*
- the nash bargaining surplus is

$$S_n^R(\underline{p}, \underline{s}) \equiv \max \left\{ \max_{b' \in \mathrm{B}_+} y(\underline{p}, x, n) - \chi_o + q(\underline{p}, x, b')b' - \delta k - \underline{w}n + \beta \cdot \frac{1}{\iota} \cdot \mathbb{E}_{x'|x} \left[V(\underline{p}, \underline{s}') \right] - R^7(\underline{p}, \underline{s}), 0 \right\} \tag{2.5.39}$$

- the wage compensation is

$$w^R(\underline{p}, \underline{s}, e, n) \cdot n = \underline{w} \cdot n + \theta_U(r) \cdot S_n^R(\underline{p}, \underline{s}) \tag{2.5.40}$$

- the expected recovery value under Ch 11 is

$$R_{e,n}^{11}(\underline{p}, \underline{s}) \equiv R^7(\underline{p}, \underline{s}) + \alpha^R(e; \theta_U(r)) \cdot \theta_C \cdot (1 - \theta_U(r)) \cdot S_n^R(\underline{p}, \underline{s}) \tag{2.5.41}$$

- the share of expected surplus that goes to shareholders

$$S_{e,n}^F(\underline{p}, \underline{s}) \equiv \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot (1 - \theta_U(r)) \cdot \iota \cdot S_n^R(\underline{p}, \underline{s}) \tag{2.5.42}$$

Accordingly, the reorganization problem (2.5.30) *can be rewritten as*

$$V^R(\underline{p}, \underline{s}) = \max_{e \in \mathrm{E}} \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot (1 - \theta_U(r)) \cdot \iota \cdot \max_{n \in \mathrm{N}} S_n^R(\underline{p}, \underline{s}) - c(e) \tag{2.5.43}$$

Proof. See Appendix 1.D.3. □

By substituting (2.5.39) in (2.5.40), the main result of Proposition 2.4.1 carries through the dynamic framework: the firm uses the threat of liquidation to reduce the bargaining position of the workers. (Again, this result stems from the timing of the restructuring problems.) The reorganization problem (2.5.30) boils down to (2.5.43): the firm chooses (n, b') to maximize the share of expected discounted value of future dividends that is not extracted by the workers, or creditors (2.5.43). Appendix 1.D.3 discusses the separability of the max operator over the firm's choices, b' and n.

Proposition 2.5.8. *The labour demand, n^*, and output, y^**

$$n^*(\underline{p},\underline{s}) = z\cdot x\cdot\left(\frac{\alpha\eta}{\underline{w}}\right)^{\frac{1}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \quad (2.5.44) \qquad y^*(\underline{p},\underline{s}) = z\cdot x\cdot\left(\frac{\alpha\eta}{\underline{w}}\right)^{\frac{\alpha\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \quad (2.5.45)$$

coincides with the ones under continuation (Proposition (2.5.3)*).*

Proof. See Appendix 1.D.3. □

The model does not capture the lay-offs of workers that firms experience during bankruptcy. Since firms and workers bargain over the wage (and not over wage and number of workers) there are no distorsions in size choices. That said, the wage compensation $\underline{w}^R(\underline{p},\underline{s},e^*,n^*)\cdot n^*$ shrinks during bankruptcy (as in the data), because of the reduction in the surplus due to the threat of liquidation. Beside tractability, this particular class of labour contracts allows me to get sharper predictions: in the model, all the inefficiencies arise from extra-ordinary decisions (extensive margin) and not from inefficient size choices (intensive margin). This has important consequences on the interpretation of the quantitative results, which should be taken as a conservative measure of the impact of pro-creditor bankruptcy reforms.
Substituting the optimal choices, we get an expression for the nash bargaining surplus in reorganization, the expected recovery value under Ch 11, and the expected share of the surplus that goes to shareholders and workers

$$S^R(\underline{p},\underline{s}) \equiv \max\left\{\max_{b'\in \mathrm{B}} y(\underline{p},x,n^*) - \chi_o + q(\underline{p},x,b')b' - \delta k - \underline{w}n^* + \beta\cdot\frac{1}{\iota}\cdot\mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right] - R^7(\underline{p},\underline{s}), 0\right\} \tag{2.5.46}$$

$$R_e^{11}(\underline{p},\underline{s}) \equiv R^7(\underline{p},\underline{s}) + \alpha^R(e;\theta_U(r))\cdot\theta_C\cdot(1-\theta_U(r))\cdot S^R(\underline{p},\underline{s}) \tag{2.5.47}$$

$$S_e^F(\underline{p},\underline{s}) \equiv \alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot(1-\theta_U(r))\cdot\iota\cdot S^R(\underline{p},\underline{s}) \tag{2.5.48}$$

$$S_e^W(\underline{p},\underline{s}) \equiv \alpha^R(e;\theta_U(r))\cdot\theta_U(r)\cdot S^R(\underline{p},\underline{s}) \tag{2.5.49}$$

Expressions (2.5.47), (2.5.48), (2.5.49) are the dynamic version of the solution to the debt restructuring and labour restructuring problem in the static model (2.4.7), (2.4.8), (2.4.9).

The Restructuring Effort Problem

The effort cost function $c:\underline{P}\times\underline{S}\times \mathrm{E}\rightarrow\mathbb{R}_+$, with $c'(\underline{p},\underline{s},\cdot)>0$, gives the shareholders' cost in firm $(\underline{p},\underline{s})$ to exert e units of effort to restructure labour contracts (in output units). Let $\alpha^R:\mathrm{E}\times\mathrm{R}\rightarrow[0,1]$, with $\alpha^{R\prime}(\cdot;r)>0$, denote the likelihood of success of Ch 11 for a given effort, e, and bargaining power of workers, $\theta_U(r)$, with $r\in\mathrm{R}\equiv\{L,H\}$. Shareholders choose the amount of effort $e\in\mathrm{E}\subseteq\mathbb{R}_+$ that solves

$$V^R(\underline{p},\underline{s}) = \max_{e\in\mathrm{E}} S_e^F(\underline{p},\underline{s}) - c(\underline{p},\underline{s},e) \tag{2.5.50}$$

where $V^R(\underline{p}, \underline{s})$ is the value of reorganization that results from the restructuring activity. Because of the symmetry between the dynamic and static model, it is easy to show that at optimum, e^*, the expected recovery value under Ch 11

$$R^{11}(\underline{p}, \underline{s}) \equiv R^7(\underline{p}, \underline{s}) + \underbrace{\alpha^R(e^*; \theta_U(r)) \cdot \theta_C}_{\text{Trade off}} \cdot (1 - \theta_U(r)) \cdot S^R(\underline{p}, \underline{s}) \tag{2.5.51}$$

carries on the main trade-off illustrated in (2.4.12): an increase in creditor rights (θ_C) ought not to increase expected recovery values.

2.5.9 The Credit Intermediaries

In the economy there is a competitive financial sector. Each risk neutral credit intermediary offers a set of firm-specific contracts $\left(\underline{p}, x, b', q(\underline{p}, x, b')\right) \in \Omega\left(\underline{p}, x, b'\right)$. Let the pricing function $q : \underline{P} \times \underline{S} \to \mathrm{Q} \equiv [0, q_{\max}] \subset \mathbb{R}_+$ be

$$q(\underline{p}, x, b') = \begin{cases} \dfrac{1}{1 + r_F} & b' \leq 0 \\[2ex] \dfrac{1}{b'(1 + r_F)} \mathbb{E}\left[\underbrace{(1 - \phi_X \cdot \phi_D)}_{\text{No Default}} \cdot b' + \underbrace{\phi_X \cdot \phi_D}_{\text{Default}} \cdot \Big(\underbrace{\phi_R}_{\text{Ch 11}} \cdot R^{11}(\underline{p}, \underline{s}) + \underbrace{(1 - \phi_R)}_{\text{Ch 7}} R^7(\underline{p}, \underline{s}) \Big) \right] & b' > 0 \end{cases} \tag{2.5.52}$$

with $\phi_i \equiv \phi_i(\underline{p}; \underline{s}')$, $i = \{X, D, R\}$. Under price competition, we have that (2.5.52) holds with equality whenever contracts are traded in strictly positive quantities[50]. So, firms earn the risk-free interest rate, r_F, on their savings, $b' \leq 0$. Conversely, loans' prices depend on the endogenous probability that the firm will meet its debt obligation and the recovery rates upon default; both these factors are function of firms' bankruptcy choices. In conclusion, let $a(\underline{p}, x, b')$ denote the aggregate amount of contracts $\left(\underline{p}, x, b', q(\underline{p}, x, b')\right)$ issued

$$a(\underline{p}, x, b') \equiv \underbrace{q(\underline{p}, x, b') \cdot b'}_{\text{Amount of Loan granted}} \left[\underbrace{\int_{\underline{P} \times \mathrm{X}} \Big[\underbrace{(1 - \phi_X)}_{\text{Continuation}} + \underbrace{\phi_X \phi_D \phi_R}_{\text{Reorganization}} \Big] \cdot \mathbb{I}_{b'(\underline{p}, \underline{s}) = b'} \, \mu(b, \mathrm{d}s)}_{\text{Measure Incumbents asking for debt}} + \underbrace{M \int_{\underline{P} \times \mathrm{X}} \phi_E \cdot \mathbb{I}_{b'(\underline{p}, x) = b'} \, G(\mathrm{d}s)}_{\text{Measure Entrants asking for debt}} \right] \tag{2.5.53}$$

[50] As a result of the optimization problem of a profit maximiser risk neutral credit intermediary with deep pockets.

Entrants

A large number of ex-ante identical potential entrants, M, decides whether to pay a fixed cost to enter, χ_E.

If they pay, they draw from the probability measure space $(\underline{P} \times \mathrm{X}, \mathcal{B}(\underline{P} \times \mathrm{X}), G_{r,k,z,x})$ a region, r, a permanent idiosyncratic productivity, z, a permanent capital scale, k, and an idiosyncratic persistent productivity shock, x.

Upon the realization (r, k, z, x), entrants decide whether to actually enter ($\phi_E = 1$) or not ($\phi_E = 0$). If they decide to enter, they finance their capital scale, k, by accessing a menu of firm specific not contingent debt contracts $\left(\underline{p}, x, b', q(\underline{p}, x, b')\right) \in \Omega\left(\underline{p}, x, b'\right)$[51] and, ultimately, by issuing equity. The problem of a potential entrant, can be described as

$$V^E(\underline{w}) = \int_{\underline{P}\times \mathrm{X}} \max_{\phi_E} \phi_E \cdot \left[\max_{(d,b')\in \mathrm{D}\times \mathrm{B}} g(d) + \beta \cdot \int_{\mathrm{X}} V(\underline{p}, \underline{s}')Q(x, \mathrm{d}x')\right] G_{r,k,z,x}(r, \mathrm{d}k, \mathrm{d}z, \mathrm{d}x)$$
$$\text{s.t. } d + k \leq q(\underline{p}, x, b') \cdot b'$$
$$g(d) = \left(\mathbb{I}_{\{d\geq 0\}} + \iota_E \cdot \mathbb{I}_{\{d<0\}}\right) \cdot d \tag{2.5.54}$$

By assuming free entry in the credit industry, the wage $\underline{w} \in \mathbb{R}^+$ is such that

$$V^E(\underline{w}) \geq \chi_E \tag{2.5.55}$$

with equality if in steady state, $M > 0$. Henceforth I will refer to (2.5.55) as the free entry condition (FEC) and to $\underline{w}$ as the FEC-wage. Differently from Hopenhayn [1992], firms finance the capital scale, k, by issuing equity or firm specific debt contracts $q(\underline{p}, x, b') \cdot b'$ that depends on the bankruptcy law. In conclusion, because of the decreasing returns-to-scale production technology and the proportional investment cost, δk, firms can finance a high capital scale, k, only if they draw a high permanent productivity, z. As a result, in equilibrium large firms have to be productive.

Invariant Distribution

Let $\Pi^I_{\underline{p}} : (\mathrm{B} \times \mathrm{X}) \times \left(2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right) \to [0, 1]$ be the transition function of a (r, k, z)-type incumbent from the state (b, x) to the state $Z \equiv Z^{b'} \times Z^{x'}$,

$$\Pi^I_{\underline{p}}((b, x), Z) = \left[1 - \phi_X(\underline{p}, \underline{s}) \cdot \left(1 - \phi_D(\underline{p}, \underline{s}) \cdot \phi_R(\underline{p}, \underline{s})\right)\right] \cdot \mathbb{I}_{b'(\underline{p},\underline{s})\in Z^{b'}} \cdot \int_{Z^{x'}} Q(x, \mathrm{d}x')$$

where $Z^{b'}, Z^{x'}$, are the projections of $Z \in \left(2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right)$.

Similarly, let $\Pi^E_{\underline{p}} : \mathrm{X} \times \left(2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right) \to [0, 1]$ be the transition function of a (r, k, z)-type

[51] Notice how the bankruptcy law affects firms entry decision by changing the feasible set of external financing opportunities.

entrant, defined as

$$\Pi^E_{\underline{p}}(x, Z) = \phi_E(\underline{p}, x) \cdot \mathbb{I}_{b'(\underline{p},x)\in Z^{b'}} \cdot \int_{Z^{x'}} Q(x, \mathrm{d}x')$$

Let μ a probability measure in the space $\Gamma\left(\mathrm{B} \times \mathrm{X}, 2^{\mathrm{B}} \times \mathcal{B}(\mathrm{X})\right)$ of probability measures. Then, I can define the operator $(\Psi\mu)$:

$$(\Psi\mu)(Z) = \sum_B \int_{\underline{P}\times\mathrm{X}} \Pi^I_{\underline{p}}((b, x), Z)\mu(\mathrm{d}\underline{p}, \mathrm{d}\underline{s}) + M \int_{\underline{P}\times\mathrm{X}} \Pi^E_{\underline{p}}(x, Z) G_{r,k,z,x}(r, \mathrm{d}k, \mathrm{d}z, \mathrm{d}x) \tag{2.5.56}$$

2.5.10 The Household

The economy is populated by a unit measure of infinitely-lived, identical households, with preferences over streams of consumption - represented by an istantaneous Bernoulli utility function $u(C)$ - that discount the future as the firms, β.

In each period each household is endowed with N_s unit of time that supplies inelastically. It further decides how much to consume, C, and how much to lend to the financial intermediaries, B'. Accordingly, the problem of the representative household can be described as

$$\begin{aligned} V_H(B; \mu) &= \max_{\{C,B'\}} u(C) + \beta \cdot V_H(B') \qquad (2.5.57) \\ \text{s.t.} \quad C + q_{\max}B' &= W + D + B \end{aligned}$$

where D is the aggregate dividend, and $q_{\max} \equiv \dfrac{1}{1 + r_F}$, where r_F is the risk free interest rate. Then in steady state

$$\beta = q_{\max} \equiv \frac{1}{1 + r_F} \tag{2.5.58}$$

2.5.11 The Aggregates of the Economy

The producing firms in the economy are the incumbents that either do not exit or reorganize. As a result, the net aggregate output

$$
\begin{aligned}
Y \equiv & \sum_B \int_{\underline{P}\times X} \left[1-\phi_X\left(1-\phi_D\phi_R\right)\right] y^*(\underline{p},\underline{s})\mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \\
& + \sum_B \int_{\underline{P}\times X} \phi_X(1-\phi_D)\cdot(1-\delta)\cdot k\ \mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \\
& - \sum_B \int_{\underline{P}\times X} \left[\underbrace{\left[1-\phi_X\left(1-\phi_D\phi_R\right)\right]\cdot\chi_o}_{\text{Maintainance cost of operation}} + \underbrace{\phi_X\phi_D\phi_R\cdot c(\underline{p},\underline{s},e^*)}_{\text{Reorganizing costs}} \right] \mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \\
& - \sum_B \int_{\underline{P}\times X} \left[1-\phi_X\left(1-\phi_D\phi_R\right)\right]\cdot\left[\iota_I-1\right]\cdot\mathbb{I}_{\{d<0\}}\ \mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \\
& - M\int_{\underline{P}\times X} \phi_E\cdot\left[\iota_E-1\right]\cdot\mathbb{I}_{\{d^*<0\}} G_{r,k,z,x}(r,\mathrm{d}k,\mathrm{d}z,\mathrm{d}x) - \underbrace{M\cdot\chi_E}_{\text{Entry cost}}
\end{aligned} \tag{2.5.59}
$$

The aggregate investment is

$$
I \equiv \sum_B \int_{\underline{P}\times X} \left[1-\phi_X(1-\phi_D\phi_R)\right]\cdot\delta k\,\mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) + M\int_{\underline{P}\times X} \phi_E\cdot k\ G_{r,k,z,x}(r,\mathrm{d}k,\mathrm{d}z,\mathrm{d}x) \tag{2.5.60}
$$

and, by national income accounting (resource constraint) aggregate, consumption is

$$
C = Y - I \tag{2.5.61}
$$

The aggregate dividends amount to

$$
\begin{aligned}
D \equiv & \sum_B \int_{\underline{P}\times X} \left[1-\phi_X\phi_D\left(1-\phi_R\right)\right] g(d^*(\underline{p},\underline{s}))\mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \\
& + M\int_{\underline{P}\times X} \phi_E g\left(d^*(\underline{p},\underline{s})\right) G_{\underline{s}}(\mathrm{d}s)
\end{aligned} \tag{2.5.62}
$$

The aggregate demand of labour equals

$$
N^d \equiv \sum_B \int_{\underline{P}\times X} n^*(\underline{p},\underline{s})\mu(\mathrm{d}\underline{p},\mathrm{d}\underline{s}) \tag{2.5.63}
$$

where $n^*(\cdot)$ is defined in (2.5.27).

Then, the aggregate demand of loans is

$$B^d \equiv \sum_B \int_{\underline{P}\times \mathrm{X}} a(\underline{p}, x, b') \ \mu(b, \mathrm{d}s) \tag{2.5.64}$$

where $a(\underline{p}, x, b')$ is defined in (2.5.53).
In conclusion, the aggregate wage equals

$$W \equiv \sum_B \int_{\underline{P}\times \mathrm{X}} w(\underline{p}, \underline{s}) \cdot n^*(\underline{p}, \underline{s})\mu(\mathrm{d}\underline{p}, \mathrm{d}\underline{s}) \tag{2.5.65}$$

2.5.12 The Equilibrium

Definition. A steady-state competitive equilibrium is a wage $\underline{w}$, a set of price schedules $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, a measure μ^*, a mass of potential entrants M^*, the incumbents policies $\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\}$, the entrants policy functions $\{\phi^{E,*}, b_e'^{,*}, d_e^*\}$, and the household decisions $(C^*, B'^{,*})$ such that:

1. given $\underline{w}$ and $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, then $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$ solve the incumbents problem (2.5.21);

2. given $\underline{w}$, $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$ and $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, then $\{\phi_E^*, b_e'^{,*}, d_e^*\}$ solve the entrants problem (2.5.54);

3. given $\underline{w}$, $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, and $B'^{,*}$, then C^* solves the household problem (2.5.57);

4. given $\underline{w}$, $\{w^*, q^*, w^{R,*}\}$, and $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, then $\alpha^{C,*}$ is the nash bargain solution (2.5.34);

5. given $\underline{w}$, $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, then q^* satisfies the zero-profit condition (2.5.52)

6. given $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, then $\underline{w}$ satisfies FEC (2.5.55);

7. given $\underline{w}$, $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, then $\mu^* = \Psi\mu^*, \forall M$;

8. given $\underline{w}$, $\{w^*, q^*, w^{R,*}, \alpha^{C,*}\}$, $\left\{\phi_X^*, \phi_D^*, \phi_R^*, b'^{,*}, n^*, d^*\right\}$, and $\{\phi_E^*, b_e'^{,*}, d_e^*\}$, μ^* and C^*:

   8.1. M^* is such that labour market clears, $N^s = N^d(M^*)$, where N^d defined in (2.5.63);

8.2. $B^{\prime,*}$ is such that the loan market clears, $B^d = B^{\prime,*}$, where B^d defined in (2.5.64).

2.6 Quantitative Analysis

How did the shift in creditor rights protection regime affect firms' bankruptcy choices, and the firms distribution? What would have happened if Ch 11 had never been introduced in 1979? To answer these questions I calibrate the dynamic model to the U.S. economy from 1979-1998. The firm level accounting data are from Compustat North-America Fundamentals Annual, 1950-2012; further information on bankruptcy are from UCLA LoPucki Bankruptcy Research Database, 1980-2012.

2.6.1 Functional Forms

The calibration requires more structure on both the uncertainty governing the model economy, and the restructuring process.

Uncertainty

The log-idiosyncratic productivity shock, $\ln x_t$, follows an AR(1) process

$$\ln x_{t+1} = (1 - \rho_{\ln x}) \cdot \bar{\mu}_{\ln x} + \rho_{\ln x} \ln x_t + \epsilon_{t+1}, \qquad \epsilon_{t+1} \sim N\left(0, \sigma^2_{\epsilon}\right) \tag{2.6.66}$$

I approximate the process with a discrete-state Markov chain, by using Gauss-Hermite nodes and weights and by applying the Tauchen and Hussey [1991] weights correction in order to account for the persistency. I discretize the support[52] of the idiosyncratic productivity shock using 9 points.

For what concerns the uncertainty at entry, $G_{r,k,z,x}$, I make the following assumptions: the permanent and persistent idiosyncratic productivity shocks are drawn from log-normal distributions, $G_z(\mu_{G(z)}, \sigma_{G(z)})$ and $G_x(\mu_{G(x)}, \sigma_{G(x)})$; the fixed capital scale is drawn from a pareto distribution $G_k((\kappa_k, k_{\min})$. In conclusion entrants are born with probability $p_{\theta_U(H)}$ in the highly unionized region (formally $G_r = \{1, 0; p_{\theta_U(H)}, 1 - p_{\theta_U(H)}\}$). By assuming independence across these dimensions, I have $G_{r,k,z,x} = G_r \cdot G_k \cdot G_z \cdot G_x$.

2.6.2 The Restructuring Effort Problem

I need to specify functional forms for the effort cost function, $c(\underline{p}, \underline{s}, e)$, and the likelihood of a success of Ch 11, $\alpha^R(e; \theta_U(r))$ (Section 2.5.8).

I assume $c(\underline{p}, \underline{s}, e)$ is linear in the surplus, $c(\underline{p}, \underline{s}, e) = c(e) \cdot \iota \cdot S^R(\underline{p}, \underline{s})$, as in the static model; differently, I set $c(e) = c_{11} \cdot e$.

[52]Following standard practice, I determine the bounds of the support using a trimming parameter $m = 20$.

Then, I set $\alpha^R(e;\theta_U(r)) = s^r_{11,p} \cdot (1 - \exp(-e) \cdot \theta_U(r))$, with $r = L, H$. An interpretation is in order. As explained in the static model, this specification formalizes the intuition that without a formal attempt to restructure labour contracts, $e = 0$, the probability of success of the reorganization procedure decreases with the bargaining power of workers, $1 - \theta_U(r)$. By exerting restructuring effort, shareholders can temper this negative effect and increase the likelihood of success, $1 - \exp(-e) \cdot \theta_U(r)$. In this context, $\theta_U(r)$ proxies for the reluctance of workers to accept changes in labour conditions, namely the reduction in the wage w^R. Lastly, I scale the likelihood of success of Ch 11 in the two regions by a different factor $s^r_{11,p}$ with $r = L, H$, to let the U.S. data say where the scope of restructuring labour contracts is stronger.

2.6.3 Calibration

The economy is calibrated over the period 1979-1998[53]. One period in the model corresponds to one year. The model has 28 parameters: physical technology (α, η, δ,χ_o), labour market ($\theta_U(L)$, $\theta_U(H)$, $p_{\theta_U(H)}$), financing technology (ι_I, ψ, θ_C), restructuring technology (c_{11}, $s^L_{11,p}$, $s^H_{11,p}$), entrants (χ_E, ι_E), discounting (β, r), labour supply, N, and uncertainty $\{(\bar{\mu}_{\ln x}, \rho_{\ln x}, \sigma_\epsilon), (\mu_{G(x)}, \sigma_{G(x)}), (\mu_{G(z)}, \sigma_{G(z)}), (\kappa_k, k_{\min}), p_X\}$, whose parameters are discussed, in details, in the next section.

I use estimates or impose restrictions on 12 parameters and structurally estimate the rest.

2.6.4 Parameters Restrictions

I start by imposing restrictions on the **uncertainty** governing the model economy.

First of all, I set the unconditional mean of the log-idiosyncratic productivity shock to 0, $\bar{\mu}_{\ln x} = 0$. Following, I impose restrictions on $(\mu_{G(x)}, \sigma_{G(x)})$, $(\mu_{G(z)}, \sigma_{G(z)})$. In particular, I assume the initial idiosyncratic productivity shocks x' are drawn from the long-run log normal distribution, $G_x(0, \sigma_\epsilon/\sqrt{1 - \rho^2_{\ln x}})$. Hence, I assume $G_z = G_x$, and I discretize the permanent idiosyncratic productivity into 3 levels associated to the conditional expectation of z falling in one of the following intervals: $[0, x_{20th}]$, $[x_{20th}, x_{80th}]$, $[x_{80th}, \infty]$, where x_{qth}, denotes the qth percentiles. By so doing I tie the cross-sectional distribution properties of the permanent efficiency with the long-run property of the efficiency process estimated in the data.

Next I move to the **physical technology**. Following Gilchrist et al. [2013], I set the value-added share of labour in the production function $\alpha = 0.7$, and the estimated decreasing return to scale parameter $\eta = 0.85$[54]. The real risk-free rate and the annual firm

[53] In 1978 the U.S. Congress enacted the Bankruptcy Reform Act, which became effective on October 1, 1979.

[54] These parameters are consistent with the literature (e.g. Barseghyan and DiCecio [2011]). In turn, this parameters specification imply a decreasing returns to scale parameter over physical capital $\gamma = 0.63$

		Physical Technology	
α	0.70	Value-added share of labour	Gilchrist et al. [2013]
η	0.85	Production Function Returns to Scale	Gilchrist et al. [2013]
		Financing Technology	
ι_E	ι_I	Equity issuance cost entrants	Restriction
		Bankruptcy Technology	
$s_{11,p}^{H}$	1.00	Scope of restructuring in highly unionized region	Normalization
		Economy	
β	0.96	Subjective Discount Factor	FOC
r^F	0.04	Real Risk-Free Interest Rate	FRED
N^s	1.00	Labour Supply	Normalization
		Uncertainty	
$\bar{\mu}_{\ln x}$	0	Unconditional mean of ln x_t	Standard
$\mu_{G(x)}$	0	Expected persistent productivity ln x_t	Restriction
$\sigma_{G(x)}$	$\sigma_\epsilon/\sqrt{1-\rho_{\ln x}^2}$	Standard deviation of persistent productivity ln x_t	Restriction
$\mu_{G(z)}$	0	Expected permanent productivity ln z_t	Restriction
$\sigma_{G(z)}$	$\sigma_\epsilon/\sqrt{1-\rho_{\ln x}^2}$	Standard deviation of permanent productivity ln z_t	Restriction

Table 2.6.4: Parameters Restrictions

discount factor are set to $r = 0.04$. By (2.5.58), the steady-state household annual discount rate is $\beta = \frac{1}{1 + r_F} = 0.9615$. Hence, I normalize the labour supply $N^s = 1$.

For what concerns the **financing technology**, I assume that entrants and incumbents face the same equity issuance cost, $\iota_I = \iota_E$.

I conclude by normalizing the scope of restructuring parameter in the highly unionized region to $s^H_{11,p} = 1$. Table 2.6.4 summarizes parameters and restrictions.

2.6.5 Estimation Strategy

I estimate 16 parameters by minimizing the weighted sum of squared residual between a set of moments computed in the model, $m(\underline{\theta})$, and in the data, $\hat{m}$. I choose 34 moments that are a priori informative[55] about the firms distribution and the phenomenon of corporate bankruptcy default.

Since some moments are more sensitive to changes in some parameters, to illustrate the tightest links, I partition the set of estimated parameters in two: the one responsible for the default/exit phenomenon, and the one responsible of the firm distribution. For what concern the parameters responsible to match the default/exit phenomenon: p_X targets the aggregate exit rate (by default and not); χ_o is used to match the aggregate default rate; ψ matches the Ch 7 default rate; $\theta_U(L)$, $\theta_U(H)$ match Ch11 default rates in the lowly and highly unionized regions; c_{11} and $s^L_{11,p}$ match the fraction of Ch 11 that are converted to Ch 7 in the highly and lowly unionized region, respectively; $p_{\theta_U(H)}$ targets the fraction of firms in highly unionized states; θ_C targets the aggregate median recovery value under Ch 11. For what concerns the firm distribution, I devote a set of parameter to capture information about the size of the firms and another set of parameters to capture moments related to the leverage: the equity issuance cost ι_I is used to match the expected leverage of the incumbents; the entry cost χ_E targets the Tobin-q statistics of incumbents; $k_{\min}$ targets the median leverage at entry; κ_k and δ matches the cross-section standard deviation of leverage and Tobin-q of incumbents. In conclusion $\rho_{\ln x}$ σ_ϵ have major effects on all the statistics in the model. In particular, I use them to match 10th, 20th, 50th, 70th, 90th percentiles of the distribution of leverage (incumbents and entrants) and Tobin-Q. Table 3.5.7 reports the results of the estimation.

2.7 The Shift in Creditor Rights Protection Regime

How did the shift in creditor rights protection regime affect the U.S. economy and the firms financial structure? In this section I use the calibrated model economy to answer

consistent with the lower bound of reasonable parameters for the class of Cobb-Douglas production function (e.g. Arellano et al. [2012]).

[55]Heuristically speaking the moments are informative about the unknown parameter if they are sensitive to its changes.

Table 2.6.5: Simulated Method of Moments Estimation

Target	**Data**	**Model**	**Parameter**		**Description**
			Default		
E_i[Exit Rate]	0.0639	0.0192	p_X	0.0148	Exogenous Prob Exit
E_i[Default Rate]	0.0077	0.0075	χ_o	6.4215	Maintenance Cost
E_i[Ch 7 Default Rate]	0.0023	0.0019	ψ	0.7672	Clearance Loss under Ch 7
E_i[Ch 11 Default Rate$\|r = L$]	0.0020	0.0015	$\theta_U(L)$	0.1192	Unions Barg.pow in region L
E_i[Ch 11 Default Rate$\|r = H$]	0.0034	0.0041	$\theta_U(H)$	0.3741	Unions Barg pow in region H
$E_i[\alpha^R(e^*)\|r = H]$	0.9440	0.9440	c_{11}	0.0110	Cost of Restructuring Effort
$E_i[\alpha^R(e^*)\|r = L]$	0.9661	0.9661	$s^L_{11,p}$	0.9681	Scope of restructuring in region L
$E_i[\mathbb{I}_{r=H}]$	0.6536	0.6224	$p_{\theta_U(H)}$	0.7113	Pr. entering in region H
$q_{i,50}[\alpha^R \cdot \alpha^C + (1-\alpha^R) \cdot C_7(k)/b]$	0.5309	0.2730	θ_C	0.4824	Creditors bargaining power
			Firms Distribution		
$q_{50,i}[B/A\,\|\,\text{Incumbents}]$	0.1360	1.2551	ι_I	1.0985	Inc. Equity Issuance Cost
$E_i[V/A\,\|\,\text{Incumbents}]$	1.6140	0.2561	χ_E	0.2531	Entry Cost
$q_{50,i}[B/A\,\|\,\text{Entry}]$	0.1190	1.2551	$k_{\min}$	0.7054	Lower Bound k
$\sigma_i[B/A\|\,\text{Incumbents}]$	0.1859	0.4494	κ_k	0.3633	Pareto Exponent G_k
$\sigma_i[V/A\,\|\,\text{Incumbents}]$	2.1469	0.3765	δ	0.2208	Depreciation Rate
$\sigma_i[B/A\,\|\,\text{Entry}]$	0.2137	0.2049	σ_ϵ	0.0943	Volatility of innovation of $\ln(x)$
$q_{50,i}[Y/\text{Employee}]$	1.4255	0.5121	$\rho_{\ln x}$	0.9657	Persistency of $\ln(x)$ AR(1)

Note: The first and second column report the structural parameters of the model and their description. The third column reports the targeted statistics: $E[\cdot]$ denotes time series averages, while $E_i[\cdot]$, $\sigma_i[\cdot]$ and $q_{x,i}[\cdot]$ denote the time series averages of, respectively, cross-sectional averages, standard deviations and cross-sectional x-percentiles. The Data column reports the moment computed in the data (firms ratios are trimmed at 1 and 99 percentiles). *Source:* Compustat North-America Fundamentals Annual, 1979-1998. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

this question. To discipline the exercise, I increase the bargaining power of creditors, θ_C, to mach the 1999-2012 fraction of Ch 11 filings that are converted to Ch 7, keeping all the other parameters fixed at their 1979-1998 levels. Table 2.7.6 reports the fit.

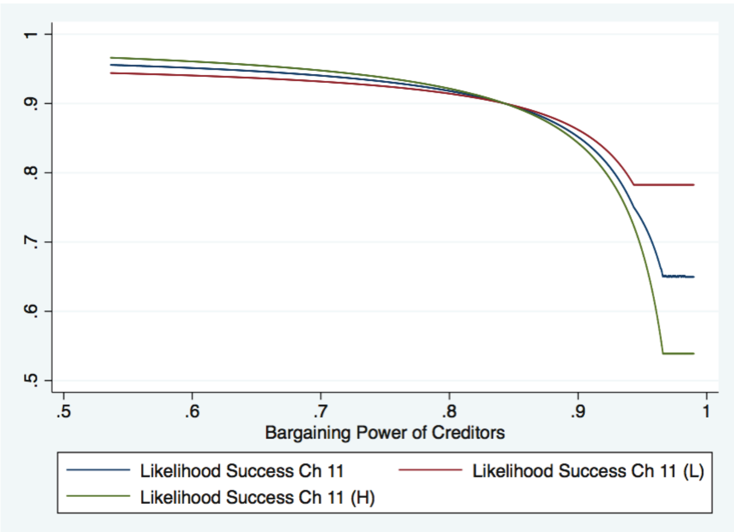
Table 2.7.6: Discipline of the increase in creditors rights protection

Panel A. *Disciplining the Shift in Creditor Rights Protection Regime*				
	1979-1998		1999-2012	
	Data	Calibrated Model	Data	Post-shift Model
θ_C	-	0.4824	-	0.7939
Likelihood of Success of Ch 11	0.9511	0.9601	0.9201	0.9129

Note: The table reports the likelihood of success (row 2) in the Data and in the Model for the pre (1979-1998) and post (1999-2012) shift period for different values of θ_C (row 1).

Table 2.7.7 compares the steady-states outcomes at region and aggregate level. As main result, it validates the mechanism. Let me start with the trigger. An increase in creditor rights depresses the restructuring effort, reducing the likelihood of success of the Ch 11 procedure (−3.5%). The effect is stronger in highly unionized states (−3.8% against −2.7%), where the scope of restructuring is stronger (see Figure 2.7.4).

Figure 2.7.4: Likelihood of Success of Ch 11 procedure and creditor rights



Note: Implied Likelihood of success of the Ch 11 procedure for different bargaining power of the creditors θ_C by unionization region: highly (green), lowly (red) and aggregate (blue). The statistic is computed using the parameterization in Tables 2.6.4 and 3.5.7, but for the bargaining power of the creditors.

Thereby, reorganization becomes less attractive than its liquidation alternative (−0.5%), especially for firms where workers extract many rents (−0.7%). Accordingly, the recovery rate upon default drops (−3.3%), driven by the drop in the recovery rates in Ch 11 (−4.88%). The cost of debt service raises and hinders firms' ability to use debt to smooth out shocks, reducing their profitability especially in highly unionized states. Firms are less likely to enter in the highly unionized region (−0.01%), and when they enter they ought to be smaller (−0.22%) and more productive. TFP increases by (0.02%) and the productivity distribution becomes more positively skewed (2.01%). Firms substitute retained earning for debt, which explains the drop in the dividend price ratio (−2.1%).

Table 2.7.7: Effect of the Shift in Creditor Rights Protection Regime

Effect of the Shift in Creditor Rights Protection Regime			
	Lowly Unionized %	Highly Unionized %	Aggregate %
Bankruptcy Composition			
Fraction of Ch 11 which are successful	-3.8566	-5.3032	-4.9218
Fraction of defaulters which file for Ch 11	-0.2207	-0.9863	-0.7591
Firms Distribution			
Assets per firm	-0.1540	-0.3124	-0.2553
Employee per firm	-0.0974	-0.1761	-0.1512
Total Factor Productivity	0.0109	0.0334	0.0238
Total Factor Productivity *Skewness*	1.2626	2.8426	2.2483
Leverage	-0.4831	-0.1270	-0.3413
Labour Productivity	-0.0000	-0.0000	-0.0000
Tobin Q	0.9025	0.9848	1.3950
Tobin Q Standard Deviation	-0.3377	0.6012	0.3678
Dividend Price Ratio	-2.4878	-3.1062	-1.9599
National Income Accounting			
Output (Y)	0.8439	-0.5062	0.0000
Consumption (C=NY-I)	0.8298	-0.5490	-0.0307

Since debt is more expensive for unit of collateral, the model captures a country level deleverage −0.22% (as in the data), but does not match the across regions dynamics. The reason is that smaller amount of debt together with more productive firms, reduces the likelihood of default (−0.33%) on a given loan, especially in lowly unionized regions. As in the data, the model predicts an increase in the Tobin-Q volatility 0.24%, but fail to produce the across-region. The model replicates qualitatively the country-wise and regional drop in the dividend-price ratio observed in the data.

In aggregate, output does not fall, but the economy records significant regional effects.

2.8 The Economic Value of the Bankruptcy Reorganization Procedure

What is the economic value of Ch 11? This question traces its roots back to 1979 - year of the enactment of the Bankruptcy Code - when Ch 11 was for the first time introduced. I address this question by investigating what would have happened if Ch 11 had never been introduced. Table 2.8.8 compares the 1979-1998 U.S. model economy with what it would have been without Ch 11.

Table 2.8.8: Effect of Shutting Down Ch 11

Effect of Shutting Down Ch 11			
	Lowly Unionized %	Highly Unionized %	Aggregate %
Bankruptcy Composition			
Fraction of Ch 11 which are successful	-0.0000	0.0000	-100.0000
Fraction of defaulters which file for Ch 11	-100.0000	-100.0000	-100.0000
Firms Distribution			
Assets per firm	-0.5169	-5.2130	-3.1409
Employee per firm	-0.0732	-2.6724	-1.6160
Total Factor Productivity	-0.0370	0.5257	0.2383
Total Factor Productivity *Skewness*	-4.3795	48.5853	23.0546
Leverage	-47.7650	-1.0651	-19.7417
Labour Productivity	-0.1185	-0.1185	-0.1185
Tobin Q	78.4584	0.1766	84.8176
Tobin Q Standard Deviation	-39.4996	15.8793	25.3063
Dividend Price Ratio	-100.1352	65.3641	-82.1180
National Income Accounting			
Output (Y)	23.3297	-14.1846	-0.1184
Consumption (C=NY-I)	24.3028	-14.8402	-0.1263

The results are striking, and this is the logic. The closure of Ch 11 pushes expected recovery values down and makes debt more expensive. An increase in the debt cost has two countervailing effects: 1) it makes more difficult for firms to smooth out shocks; 2) it reduces the value of being an incumbent (in both regions) pushing the FEC-wage down (−0.12%). Firms in different regions are more sensitive to one or the other effect. An increase in the debt cost washes out, unproductive firms, increasing aggregate TFP (0.24%). TFP increases in highly unionized states by half a percentage point while it drops in lowly unionized states (−0.03%). The reason is that highly unionized firms suffer disproportionally more the loss of Ch 11. They experience a significant reduction in size (−5.2%) and need to be more productive to stay. The lowly unionized firms benefit more from the drop in the FEC-wage, allowing more unproductive firms to stay. This drives up the debt cost (−2.11% drop in the price), decreases their leverage, and yields a significant change in the dividend distribution policy (drop in dividend price ratio). The equity issuance becomes more attractive than the debt alternative (44%), especially for entry firms (153%). Through entry, firms relocate in the lowly unionized region. Fixing the mass of firms at entry, because of the churning effect there will be less firms, more productive, producing a greater amount of output. Then, to maintain labour demand equal to labour supply the mass of firms has to increase. Consumption and output sensibly falls by 0.1% in aggregate but with a strong asymmetric impact on the economy: it drops by 14% in the highly unionized region and increase by 24% in the lowly unionized one.

On the top of that, there are significant changes in the corporate structure of firms. The increase in the debt price comports a signficant deleveraging, extremely pronounced in lowly unionized regions (−48% vs −1%). Per unit of assets, firms are more valuable (84% increase in Tobin-Q), especially in lowly unionized regions (78%). Besides, the dividend yield drops by 80%, led by lowly unionized firms (−100%). Conversely, firms in highly unionized states experience a 65% increase.

2.9 Conclusions

In this paper I study from a positive and normative point of view the macroeconomic implications of bankruptcy reforms when workers extract rents. By doing so, I make four contributions.

First, I foreground a *channel* through which pro-creditor bankruptcy reforms can backfire, which does not appeal to agency frictions. Firms file for bankruptcy reorganization not only to restructure debt but also to restructure labour contracts. An increase in creditor rights suffocate the incentives of the shareholders to bargain with the workers, making the procedure more likely to fail. When workers extract many rents - and restructuring labour contracts is required to re-establish the economic soundness - the drop in the likelihood of success can offset the increase in recovery values upon success, and make the reform backfire.

Second, I embed the *restructuring channel* into a static model - where I use the bankruptcy law to microfound the enforcement constraint - and show how the optimal (output maximizing) level of creditor rights decreases with the bargaining power of workers. The exercise sheds some light on why more unionized countries - as Italy, France - have lower creditor rights protection than less unionized ones - say, U.S.

Third, I establish the mechanism in the U.S. data. To do that, I exploit two sources of variation: historical differences in the degree of unionization across states, and a shift in the creditor rights protection regime. As a result, I document a break in the relative use of Ch 11 in 1998, associated with a drop in the likelihood of success of Ch 11, a significant deleveraging (-27%), drop in the dividend yields (46%), a three-fold increase in Tobin-Q dispersion. The theory rationalizes the different response of highly and lowly unionized firms.

Fourth, I perform a *positive* analysis. First of all, I build a general equilibrium firm dynamic model, where the default option captures salient features of the U.S. corporate bankruptcy law. The novel ingredient is the restructuring problem among the stakeholders: shareholders, bondholders and workers. Second, I calibrate the model to the U.S. economy from 1979-1998, using firm level accounting data from Compustat North-America Fundamentals Annual , bankruptcy information from UCLA LoPucki Bankruptcy Research Database , and a proxy for the bargaining power of workers from Union Membership and Coverage database (CPS).

Then, I perform two policy experiments. In the first experiment, I use the model economy to assess the effect of the observed increase in creditor rights protection. An increase in creditor rights tempers the shareholder incentives to restructure labour contract, reducing the likelihood of success of Ch 11. In turn, it makes Ch 11 less attractive than Ch 7, causing the inefficient liquidation of viable firms. The reduction in the expected recovery rates upon default, yields an increase in the cost of debt service and a decrease in the leverage. These effects are stronger in highly unionized regions, where restructuring labour contract is more crucial for the success of the reorganization process. In a second policy experiment, I try to attach an economic value to Ch 11 gauging the losses of shutting it down. Indeed, the reorganization procedure was a novelty of the 1979 bankruptcy code. What would have happened if Ch 11 had never been introduced? Despite output and consumption do not show significant changes in aggregate (-0.11% and -0.12%) the regional effects are economically important. Highly unionized firms suffer disproportionally from the loss of the Ch 11 procedure, as summarized by a 15% drop in output and consumption. Since there are more highly unionized firms in the economy the wage that clears the free entry conditions drops. Firms in lowly unionized states benefit significantly from the drop in the wage, bringing about a significant restructuring of their financial structure (43% drop in leverage, associated with a huge decrease in dividend yield). All together the lowly unionized region records a significant increase in consumption and output (around 24%).

The quantitative results are a conservative measure of the macroeconomic implications of changes in the corporate bankruptcy law. Bankruptcy reforms affect directly extensive margin decisions (entry and form of exit), and only *indirectly* - through prices - firms' intensive margin choices (leverage, hirings,...).

In my future work, I plan to fill this gap by exploring the following extensions. On one side, by assuming that in reorganization share-holders can reduce the bargaining power of workers for a stochastic number of periods, we will observe a strategic use of leverage to enter Ch 11 default and restructure labour contracts. This mechanism would provide an alternative explanation of the strategic use of capital structure to lower hiring cost (complementing the *bargaining* channel of Quadrini and Sun [2015]). On the other side, by assuming hiring and firing costs, I can explore the interaction between my *restructuring* channel and the *bargaining* channel of Quadrini and Sun [2015]. I expect both these extensions to amplify the real effects of bankruptcy reforms.

Appendix

1.A Appendix: Data

The firm level accounting information is from COMPUSTAT North America fundamentals annual data. The cleansing of the database is conducted at several layers. Firstly, I purge the sample from utilities (NAICS 22), financial (NAICS 52) and public administration corporations (NAICS 92). Secondly, I drop CUSIPs for American Depository Receipts (ADRs)[56].

1.A.1 Description

The description of the variable is organized in three layers: firm (t, i), state (t, s) and aggregate level t. In case of user-defined variables (as instance, real debt $b_{t,i} = l_{t,i}/P_t$) the reader can find the definition of the variables in the relative subsections.

Firm Level

Sales $sale_{t,i}$. This item represents gross sales (the amount of actual billings to customers for regular sales completed during the period) reduced by cash discounts, trade discounts, and returned sales and allowances for which credit is given to customers, for each operating segment. Variable name in Compustat: sale.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: `http://wrds-web.wharton.upenn.edu/wrds/`.
Firms real output $y_{t,i} = sale_{t,i}/P_t$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: `http://wrds-web.wharton.upenn.edu/wrds/`.
Long-term debt $l_{t,i}$. (U.S. and Canadian GAAP Definition) The item represents debt obligations due more than one year from the company's balance sheet date. This item is a component of Total Liabilities (LT). This item includes: Purchase obligations and payments to officers, when listed as long-term liabilities; Notes payable, due within one year and to be refunded by long-term debt when carried as a non-current liability; Long-term lease obligations (capitalized lease obligations); Industrial revenue bonds; Advances to finance construction; Loans on insurance policies; Indebtedness to affiliates; Bonds, mortgages, and similar debt; All obligations that require interest payments; Publishing companies' royalty contracts payable Timber contracts for forestry and paper; Extractive industries' advances for exploration and development; Production payments and advances for exploration and development. This item excludes: Subsidiary preferred stock, included in Minority Interest; The current portion of long-term debt, included in Current Liabilities; Accounts payable due after one year, included in Liabilities Other; Accrued interest on long-term debt, included in Liabilities Other; Customers' deposits on bottles, kegs, and cases, included in Liabilities Other; Deferred compensation; Long-term debt should be reported net of premium or discount. Standard & Poor's will collect the net figure. Variable name in Compustat: dltt.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: `http://wrds-web.wharton.upenn.edu/wrds/`.
Real debt $b_{t,i} = l_{t,i}/P_t$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: `http://wrds-web.wharton.upenn.edu/wrds/`.
Total Asset $at_{t,i}$. This item represents the total assets/liabilities of a company at a point in time. If the company does not report a useable amount, this data item will be left blank. Variable name in Compustat: at.
Source: Compustat North-America Fundamentals Annual, 1950-2012.

[56] ADRs are securities created to permit the trading in U.S of stock listed on foreign stock exchanges.

Website: http://wrds-web.wharton.upenn.edu/wrds/.
Real asset $a_{t,i} = at_{t,i}/P_t$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Employee $n_{t,i}$. This item represents the actual number of people employed by the company and its consolidated subsidiaries. Variable name in Compustat: emp.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Output per Worker $y_{t,i}/n_{t,i} = y_{t,i}/n_{t,i}$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Probability that a Ch 11 case is converted to Ch 7, $1 - \hat{\alpha}^R = \frac{\#(\text{Ch 11 to Ch7})}{\#\text{Ch 11}}$. It is ratio of the Ch 11 filings which are converted to Ch 7 over the total number of Ch 11 filings which are not dismissed. The numerator includes Ch 11 cases which are confirmed and eventually converted. The sample includes all the Ch 11 cases which have been *disposed*[57] before the end of 1998. The data-set is purged by involuntary filings, prepackaged cases, dismissals, and missing data. The Lopucki variables involved in the computation are: Disposition, Chapter, Voluntary, Prepackaged, YearDisposed.
Source: UCLA LoPucki Bankruptcy Research Database, 1980-1998.
Website: http://lopucki.law.ucla.edu.
Average Recovery Rate under Ch 11., $\mathbb{E}[\alpha^R \cdot \alpha^C + (1 - \alpha^R) \cdot C_7(k)]$. I compute the recovery rate under Ch 11 for each case as the ratio between: a) numerator: the sum between the distribution to all classes of secured and unsecured credtors, and b) denominator: secured and unsecured creditors claims, as reported in the disclosure statement. Hence I average this statistic across cases in the same year, and compute the final statistic as time-series average of the cross-sectional first moments. The sample includes all the Ch 11 cases which have been *disposed* before the end of 1998. The data-set is purged by involuntary filings, prepackaged cases, dismissals, and missing data. On the top of that I trim all the observations for which any of the secured/unsecured claims and dispositions were missing. The Lopucki variables involved in the computation are: Chapter, Voluntary, prepackpreneg, YearDisposed, DistribUnsec, DistribSecDiscloState, ClaimsSecDiscloState, ClaimsUnsec.
Source: UCLA LoPucki Bankruptcy Research Database, 1980-1998.
Website: http://lopucki.law.ucla.edu.
Fraction of firms in highly unionized states, $\mathbb{E}[\hat{m}(p_{\theta_U(H)})]$. The statistic is computed as the 1979-1998 time series average of the percentage firms in highly unionized states.
Source: Compustat North-America Fundamentals Annual, 1979-1998.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Fraction of Ch 11 cases over total default, by region, $\mathbb{E}[\hat{m}(\theta_U(L))]$, $\mathbb{E}[\hat{m}(\theta_U(H))]$. The statistic is the 1979-1998 time series average of the ratio between the number of Ch 11 cases and the total default (Ch 7 + Ch 11), by region.
Source: Compustat North-America Fundamentals Annual, 1979-1998.
Website: http://wrds-web.wharton.upenn.edu/wrds/.

Sector Level

Sector j. NAICS classification sectors. Excluded: utilities (22), financial (52) and public administration corporations (92). Variable name in Compustat: naics.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Employment Share $Empshare_{t,s,j} = \int \phi_{\mathcal{J}(i)=j,S(i)=s} \frac{n_{t,i}}{N_{t,s}} \mathrm{d}i$, where $\mathcal{J}(i)$ and $S(i)$ are the sector and state of firm i.
Source: Compustat North-America Fundamentals Annual, 1950-2012.

[57] Using the filing year produces insignificant changes.

Website: http://wrds-web.wharton.upenn.edu/wrds/.

State Level

Union Coverage $Cov_{t,s} = N^{Cov}_{t,s}/N^{Tot}_{t,s}$. **a)** $N^{Tot}_{t,s}$ represents the all employed civilian wage and salary workers, ages 16 and over in the Current Population Survey. Not included are employed 14-15-year-olds, self-employed workers, or a small number of unpaid family workers. **b)** $N^{Cov}_{t,s}$ is the number of employed civilian wage and salary workers who has answered yes to one of these successive questions related to their principal job: A) 'On this job, is …a member of a labor union or of an association similar to a union?'. If the answer is 'no' than the worker is asked: B) 'On this job, is …covered by a union or employee association contract?'. Hence, workers are counted as covered by a collective bargaining agreement if they are union members or if they are not members but say they are covered by a union contract.
Source: Union Membership and Coverage database (CPS), 1983-2014.
Website: http://www.unionstats.com.
Number of Firms $I_{t,s}$. Total number of firms active at time t in state s.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Herfindhal Index $H_{t,s} = \int Empshare^2_{t,s,j} \mathrm{d}j$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.

Country Level

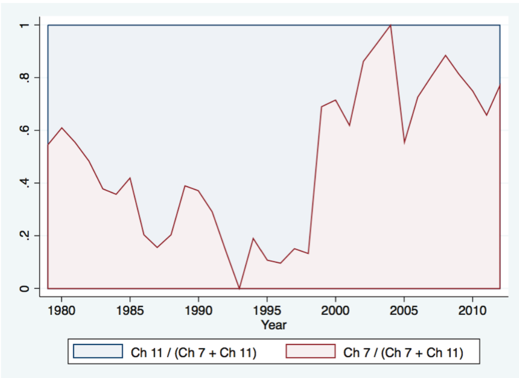
Aggregate Output $Y_{t,i} = \int_i y_{t,i} \mathrm{d}i$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Employment $N_{t,i} = \int_i n_{t,i} \mathrm{d}i$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.
Labour Productivity $Y_{t,i}/N_{t,i}$.
Source: Compustat North-America Fundamentals Annual, 1950-2012.
Website: http://wrds-web.wharton.upenn.edu/wrds/.

1.B Appendix: Empirical Analysis

1.B.1 Identification of the break

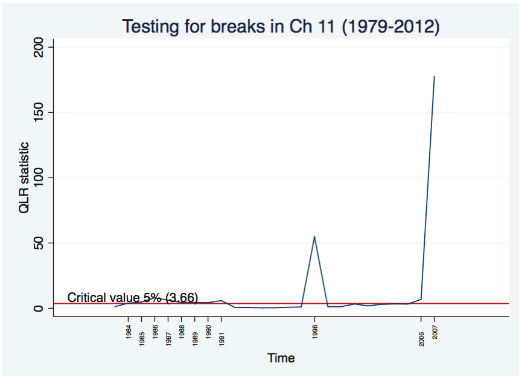
Figure 1.B.1 reports the relative use of Ch 11 procedure, computed as the ration between the annual filings for Ch 11 over the annual filings for bankruptcy by publicly listerd firms. The figure suggests a break in the relative use of the reorganization procedure in 1998. The Quandt-likelihood-ratio test - for the presence of a structural break at an unknown date in the number of annual Ch 11 filings - corroborates the finding (Fig. 1.B.2).

Figure 1.B.1: Time series of default composition by bankruptcy procedure



Note: The shaded areas denote the share of annual bankruptcy filings by bankruptcy procedure: Ch 7 (red), Ch 11 (blue). *Source:* Compustat North-America Fundamentals Annual, 1950-2012. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

Figure 1.B.2: Quandt likelihood ratio over 1979-2012



Note: QLR test - Quandt,1960. *Source:* Compustat North-America Fundamentals Annual, 1950-2012. The sample excludes: utilities (NAICS 22) financial (NAICS 52) and public administration (NAICS 92) corporations, American Depository Receipts (ADR).

1.B.2 Stability of the unionization coverage ranking over time.

Figure 1.B.3 reports the time series of the cross-sectional mean and standard deviation of the U.S. States unionization coverage, over the period 1983-2012. While the average unionization coverage has significantly decreased over time (red line), the standard deviation of coverage has remained stable (blue line). This empirical evidence suggests that the cross-sectional long-run unionization coverage rankings was preserved over time.

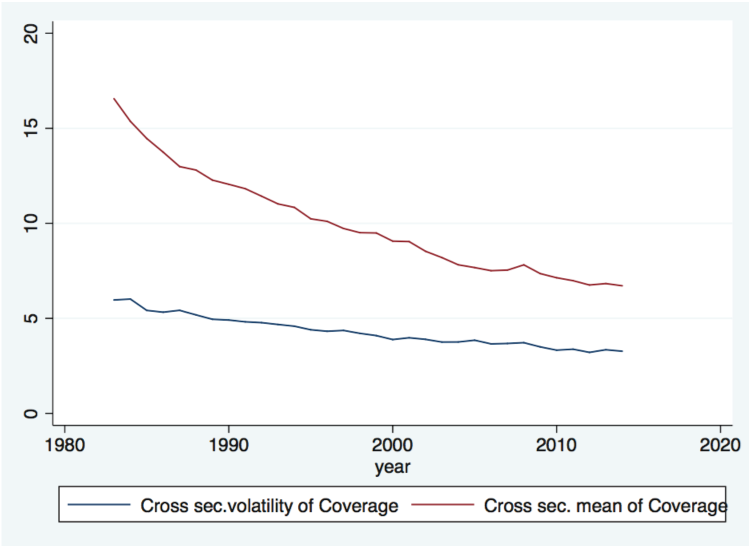
1.B.3 State-Level Analysis

See Online Appendix.

1.B.4 Firm-Level Analysis

See Online Appendix.

Figure 1.B.3: Time series of cross-sectional mean and standard deviation of Coverage



Source: Union Membership and Coverage database (CPS), 1983-2014.

1.B.5 How are results sensitive to the structural break date?

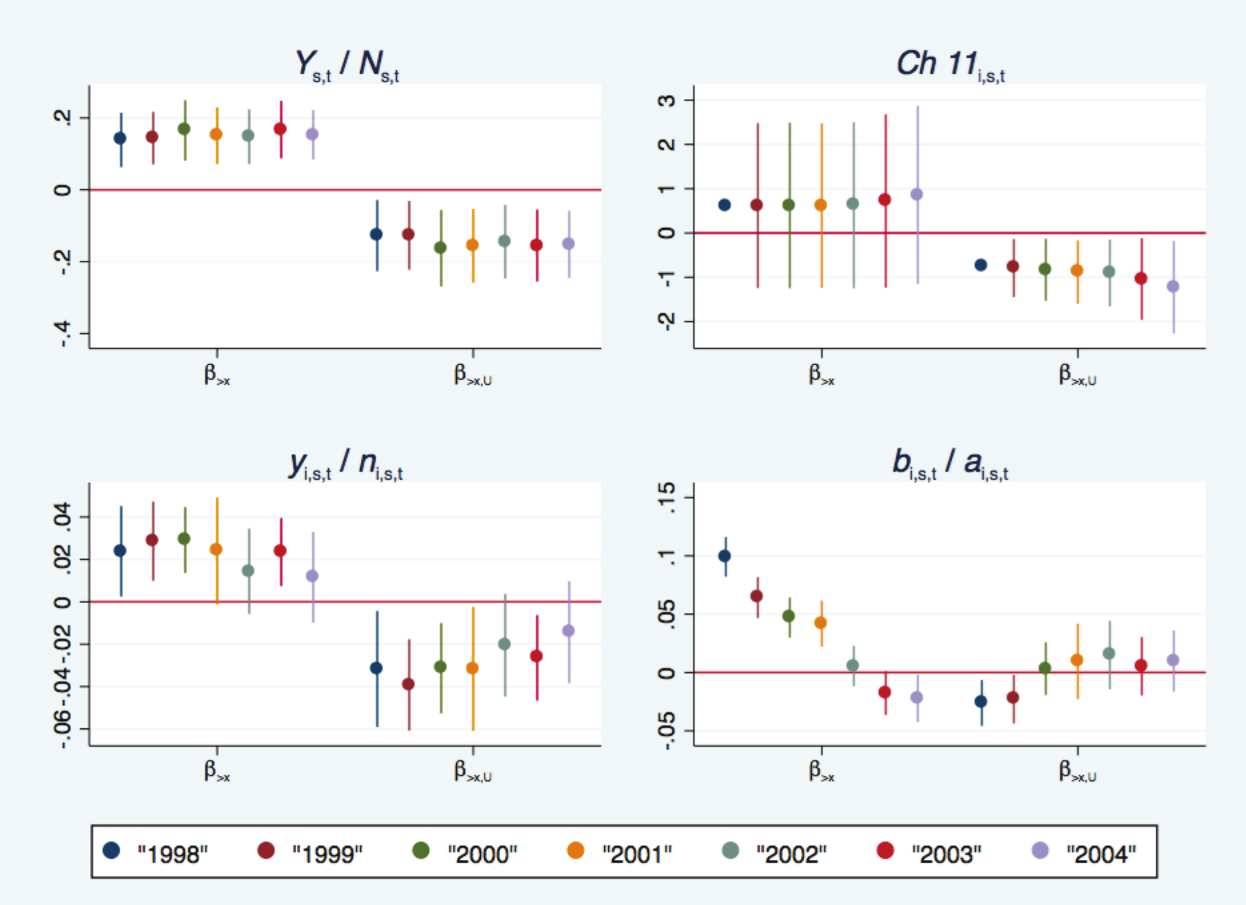
The bankruptcy literature agrees that Ch 11 looks nowadays more creditor-friendly than it did 30 years ago. Nonetheless, the shift in creditor rights protection did not arise from an amendment to the Bankruptcy Code, but from a series of causes. Warren [1999] and Miller [2007] point at financial institutions lobbying for their bankruptcy agenda. Adlera et al. [2010] identify a break in 2001, with a change in the Uniform Commercial Code ("UCC") and the adoption of UCC §9-104, that sanctioned the practice of writing control provisions into debt instruments, allowing in case of distress to shift control over a debtor's financial decisions from equity-appointed management to the creditor. Adlera et al. [2010] and Gennaioli and Rossi [2010] also suggests a shift in the judicial attitude. The enactement in 2004 of the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) contains several pro-creditor provisions relating to Ch 11 reorganizations[58]. For this reason Figure 1.B.4 reports the main coefficients of interest (break $\beta_{>1998}$ and interaction term $\beta_{>1998,U}$ in Table (2.3.3)) when the break date ranges between 1998 and 2004. Broadly speaking, results hold through.

1.B.6 How are results sensitive to the partition of firms in highly and lowly unionized?

In order to answer this question, Figure 1.B.5 reports the main coefficients of interest (break $\beta_{>1998}$ and interaction term $\beta_{>1998,U}$ in Table (2.3.3)) under different percentiles of the unionization coverage distribution separating lowly from highly unionized states: 20%, 25%, 33.33%, 50%, 66.66%, 75%, 80%. Broadly speaking, results hold through.

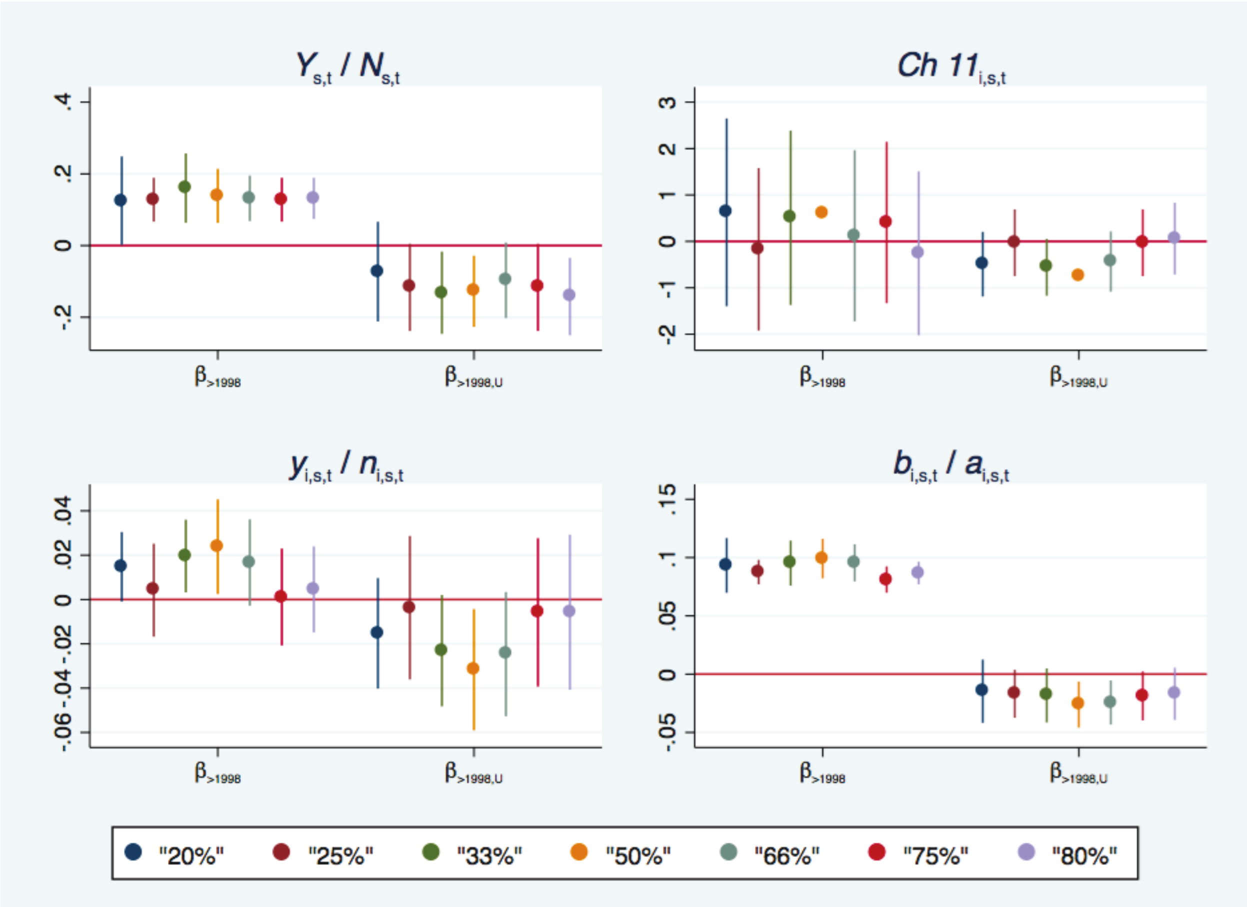
[58] Among others, the mandatory cap on a debtor's exclusive period to file a plan of reorganization; enhanced protections for reclamation and trade creditors; a mandatory cap on the period to assume or reject unexpired leases of non-residential real property; expanded protection of utilities; mandatory appointment of a chapter 11 trustee in certain circumstances and relaxation of the ability to recover preferences.

Figure 1.B.4: $\beta_{>1998,U}$, and $\beta_{>1998}$ for different definitions of $d_{>t}$



Note: Coefficient estimates and 95% c.i. on $\beta_{>1998,U}$, $\beta_{>1998}$ when $d_{>t}$ is computed at structural break dates, t: 1998, 1999, 2000, 2001, 2002, 2003, 2004. The results are displayed in 4 panels. Each panel reports - in order / by colour - the pair of coefficient estimates of $\beta_{>1998}$ (on the left) and $\beta_{>1998,U}$ (on the right) coming from the same regression. In a panel, regressions differ by the assumption on the break date. In order (by colour): 1998 (blue), 1999 (purple), 2000 (green), 2001 (orange), 2002 (light-green), 2003 (red), 2004 (violet). The 4 panels report clockwise and starting from the north-west corner coefficients estimates from: **I. State-Level Labour Productivity** Blundell-Bond two-steps regression of state level labour productivity, $\ln Y_{t,s}/N_{t,s}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit). Time and states fixed effects are reported. *Instruments:* a) GMM type: up to 3 lags of the dependent variable and continuous covariates; b) iv-type: d_U, $d_{>1998}$, and $d_{>1998} \cdot d_U$. **II. Bankruptcy Choice** Multinomial logit regressions of firms continuation choice ϕ = {Continuation, Ch 7, Ch 11} over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *Country level controls*: level of union coverage, Cov_t; 3) *State level controls*: aggregate real sales, $Y_{t,s}$, Herfindhal index of sectoral concentration (sector real sales shares unit), employment shares by sector (naics), level of union coverage, $Cov_{t,s}$, interaction terms: $d_U \cdot Cov_{t,s}$, $d_{>1998} \cdot Cov_{t,s}$, $d_{>1998} \cdot d_U \cdot Cov_{t,s}$; 4) *Firms level controls*: real sales, $y_{t,i}/P_t$, leverage, $b_{t-1,i}/a_{t-1,i}$, real total assets, $a_{t,i}/P_t$; 5) *Fixed effect controls*: time, states, and sector fixed effects are reported.. Ch 7 and Ch 11 denote the relative bankruptcy choices (the baseline case Continuation is omitted). **III. Leverage** Fixed effect regression of $\ln b_{t,i}/a_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: lagged value, $\ln b_{t-1,i}/a_{t-1,i}$, log labour productivity, $\ln y_{t,i}/n_{t,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. Leverage is measured as total liabilities over total assets (compustat identifiers: lt, at). **IV. Labour Productivity** Fixed effect regression of $\ln y_{t,i}/n_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: log labour productivity, $\ln y_{t-1,i}/n_{t-1,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. *Source:* Compustat North-America Fundamentals Annual, 1979-2012. Union Membership and Coverage database (CPS), 1983-2014.

Figure 1.B.5: $\beta_{>1998,U}$, and $\beta_{>1998}$ for different definitions of d_U



Note: Coefficient estimates and 95% c.i. on $\beta_{>1998,U}$, $\beta_{>1998}$ for different percentiles of the unionization coverage distribution separating lowly from highly unionized states: 20%, 25%, 33.33%, 50%, 66.66%, 75%, 80%. The results are displayed in 4 panels. Each panel reports - in order / by colour - the pair of coefficient estimates of $\beta_{>1998}$ (on the left) and $\beta_{>1998,U}$ (on the right) coming from the same regression. In a panel, regressions differ by the assumption on the percentile of the unionization coverage distribution separating lowly from highly unionized states. In order (by colour): 20% (blue), 25% (purple), 30% (green), 50% (orange), 70% (light-green), 75% (red), 80% (violet). The 4 panels report clockwise and starting from the north-west corner coefficients estimates from: **I. State-Level Labour Productivity** Blundell-Bond two-steps regression of state level labour productivity, $\ln Y_{t,s}/N_{t,s}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit). Time and states fixed effects are reported. *Instruments:* a) GMM type: up to 3 lags of the dependent variable and continuous covariates; b) iv-type: d_U, $d_{>1998}$, and $d_{>1998} \cdot d_U$. **II. Bankruptcy Choice** Multinomial logit regressions of firms continuation choice ϕ = {Continuation, Ch 7, Ch 11} over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *Country level controls*: level of union coverage, Cov_t; 3) *State level controls*: aggregate real sales, $Y_{t,s}$, Herfindhal index of sectoral concentration (sector real sales shares unit), employment shares by sector (naics), level of union coverage, $Cov_{t,s}$, interaction terms: $d_U \cdot Cov_{t,s}$, $d_{>1998} \cdot Cov_{t,s}$, $d_{>1998} \cdot d_U \cdot Cov_{t,s}$; 4) *Firms level controls*: real sales, $y_{t,i}/P_t$, leverage, $b_{t-1,i}/a_{t-1,i}$, real total assets, $a_{t,i}/P_t$; 5) *Fixed effect controls*: time, states, and sector fixed effects are reported.. Ch 7 and Ch 11 denote the relative bankruptcy choices (the baseline case Continuation is omitted). **III. Leverage** Fixed effect regression of $\ln b_{t,i}/a_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: lagged value, $\ln b_{t-1,i}/a_{t-1,i}$, log labour productivity, $\ln y_{t,i}/n_{t,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. Leverage is measured as total liabilities over total assets (compustat identifiers: lt, at). **IV. Labour Productivity** Fixed effect regression of $\ln y_{t,i}/n_{t,i}$ over: 1) *Treatment variables*: structural break, $d_{>1998}$, unionization dummy, d_U, interaction term, $d_{>1998} \cdot d_U$; 2) *State level controls*: Herfindhal index of sectoral concentration (sector real sales shares unit); 3) *Sector level controls*: aggregate amount of debt over aggregate amount of assets by sector in the state of consideration, $\ln B_{s,j,t}/A_{s,j,t}$, employment share of labour by sector in the state of consideration; 4) *Firms level controls*: log labour productivity, $\ln y_{t-1,i}/n_{t-1,i}$, log real total assets, $\ln a_{t-1,i}/P_t$. 5) *Other*: linear trend, t, and interaction term $t \cdot d_U$. *Source:* Compustat North-America Fundamentals Annual, 1979-2012. Union Membership and Coverage database (CPS), 1983-2014.

1.C Appendix: Static Model

The Debt Restructuring

The debt restructuring problem (2.4.2) can be rewritten as

$$NB^C_{v,e}(\underline{k}) = \alpha^R(e;\theta_U)\cdot \max_{r\in\mathbb{R}^+}[\zeta\cdot\bar{k}-v-r]^{1-\theta_C}\cdot[r-(1-\psi)\cdot\bar{k}]^{\theta_C} \tag{1.C.1}$$

$$\text{s.t}\qquad \zeta\cdot\bar{k}-v-r\geq 0 \qquad r\geq(1-\psi)\cdot\bar{k} \tag{1.C.2}$$

Upon success, the recovery value under Ch 11 which solves the problem is

$$r^* = \max\{\ \theta_C\cdot(\zeta\cdot\bar{k}-v)+(1-\theta_C)\cdot(1-\psi)\cdot\bar{k}\ ,\ \ (1-\psi)\cdot\bar{k}\ \}$$

Substituting in the objective function, it is easy to show that for a given v and e the expected surplus of the firm is

$$\begin{aligned}
S^F_{v,e}(\bar{k}) &= \alpha^R(e;\theta_U)\cdot(\zeta\cdot\bar{k}-v-r^*)\\
&= \alpha^R(e;\theta_U)\cdot\left[\zeta\cdot\bar{k}-v-\theta_C\cdot(\zeta\cdot\bar{k}-v)-(1-\theta_C)\cdot(1-\psi)\cdot\bar{k}\right]\\
&= \alpha^R(e;\theta_U)\cdot\left[(1-\theta_C)\cdot(\zeta\cdot\bar{k}-v)-(1-\theta_C)\cdot(1-\psi)\cdot\bar{k}\right]\\
&= \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\left[\zeta\cdot\bar{k}-v-(1-\psi)\cdot\bar{k}\right]
\end{aligned}$$

$$S^F_{v,e}(\bar{k}) = \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\max\left[\zeta\cdot\bar{k}-v-(1-\psi)\cdot\bar{k}\ ,\ 0\right] \tag{1.C.3}$$

Similarly, the expected surplus of the lenders is

$$S^C_{v,e}(\bar{k}) = \alpha^R(e;\theta_U)\cdot\theta_C\cdot\max\left[\zeta\cdot\bar{k}-v-(1-\psi)\cdot\bar{k}\ ,\ 0\right]$$

and the expected recovery value under Ch 11 is

$$R^{11}_{v,e}(\bar{k}) = (1-\psi)\cdot\bar{k}+\alpha^R(e;\theta_U)\cdot\theta_C\cdot\max\left[\zeta\cdot\bar{k}-v-(1-\psi)\cdot\bar{k}\ ,\ 0\right] \tag{1.C.4}$$

Equations (2.4.4), (2.4.3) follows.

The Labour Restructuring

Substituting the surplus of the firm (2.4.4) in (2.4.5) the problem reads

$$NB^U_e(\bar{k}) = \alpha^R(e;\theta_U)\cdot(1-\theta_C)^{1-\theta_U}\max_{v\in\mathbb{R}^+}[\zeta\cdot\bar{k}-(1-\psi)\cdot\bar{k}-v]^{1-\theta_U}\cdot[v]^{\theta_U}$$

from which we get that the wage compensation (2.4.6),

$$w(\bar{k}) = \theta_U\cdot\max\left[\zeta-(1-\psi)\ ,\ 0\right]\cdot\bar{k}$$

For the ease of notation, let

$$S(\bar{k}) = \max\left[\zeta-(1-\psi)\ ,\ 0\right]\cdot\bar{k}$$

denote the surplus of the firm.

Let $[\zeta-(1-\psi)]\cdot\bar{k}=0$, then $w(\bar{k})=0$ and therefore $S^F_e(\bar{k})=S^W_e(\bar{k})=0$ and $R^{11}_{v,e}(\bar{k})=(1-\psi)\cdot\bar{k}$.

On the other hand, let $[\zeta-(1-\psi)]\cdot\bar{k}>0$. Then by substituting (2.4.6) in the objective function, we have that

$$\begin{aligned}
S^F_e(\bar{k}) &= \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\max\left[\zeta\cdot\bar{k}-w(\bar{k})-(1-\psi)\cdot\bar{k}\ ,\ 0\right]\\
&= \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\max\left[[\zeta-(1-\psi)]\cdot\bar{k}-\theta_U\cdot[\zeta-(1-\psi)]\cdot\bar{k}\ ,\ 0\right]\\
&= \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot(1-\theta_C)\cdot[\zeta-(1-\psi)]\cdot\bar{k}
\end{aligned}$$

and similarly

$$S_e^W(\bar{k}) = \alpha^R(e;\theta_U)\cdot(1-\theta_C)\cdot\theta_U\cdot[\zeta-(1-\psi)]\cdot\bar{k}$$

$$R_e^{11}(\bar{k}) = (1-\psi)\cdot\bar{k} + \alpha^R(e;\theta_U)\cdot\theta_C\cdot(1-\theta_U)\cdot[\zeta-(1-\psi)]\cdot\bar{k} \tag{1.C.5}$$

The results (2.4.8), (2.4.9), and (2.4.7) follow.

1.C.1 The Restructuring Effort Problem

Proof Proposition 2.4.2

Proof. Given

$$e^* = (1-\theta_C)\cdot(1-\theta_U)\cdot\frac{\theta_U}{c_{11}}$$

Result a. The optimal level of effort decreases in θ_C

$$\frac{\partial e^*}{\partial\theta_C} = -(1-\theta_U)\cdot\frac{\theta_U}{c_{11}} < 0$$

Result b.

$$\frac{\partial e^*}{\partial\theta_U} = \frac{1-\theta_C}{c_{11}}\cdot\frac{\partial(\theta_U-\theta_U^2)}{\partial\theta_U} = \frac{1-\theta_C}{c_{11}}\cdot[1-2\cdot\theta_U]$$

The results follows. □

Substituting (2.4.10), the probability of success of the Ch 11 procedure

$$\alpha^R(e^*;\theta_U) = (1-\theta_U)\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)-\frac{\theta_U^2}{c_{11}}\cdot\theta_C\right]$$

Substituting $\alpha^R(e^*;\theta_U)$ and simplifying we get

$$\begin{aligned} R^{11}(\bar{k}) &= (1-\psi)\cdot\bar{k} + (1-\theta_U)\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)-\frac{\theta_U^2}{c_{11}}\cdot\theta_C\right]\cdot\theta_C\cdot(1-\theta_U)\cdot S(\bar{k}) \\ &= (1-\psi)\cdot\bar{k} + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C-\frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right]\cdot S(\bar{k}) \end{aligned}$$

1.C.2 Characterization of the Equilbrium

The enforcement constraint

By using (2.4.12), the recovery value in bankruptcy $L(\bar{k}) = \max[\, R^{11}(\bar{k})\,,\, R^7(\bar{k})\,]$ becomes

$$L(\bar{k}) = \left\{(1-\psi) + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C-\frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right]\cdot\max[\,\zeta-(1-\psi)\,,\,0\,]\right\}\cdot\bar{k} \tag{1.C.6}$$

The participation constraint, requires the debt repayment $R_k\cdot k$ to be not larger than the expected recovery value,

$$R_k \cdot k \leq L(\bar{k})$$

By price competition in the credit market, $R_k = 1$. Then, since the firm preferences are increasing in k, ex-*ante*, the optimal amount borrowed by the firm is

$$k^* = \min\{\, L(\bar{k})\,,\,\bar{k}\,\}$$

where the minimum operator captures the resource feasibility constraint (the lenders cannot lend more than the total amount of capital they have).
Substituting (1.C.6) we get the optimal level of borrowing

$$k^* = \min\left\{ (1-\psi) + (1-\theta_U)^2 \cdot \left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C - \frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right]\cdot \max\left[\,\zeta-(1-\psi)\,,\,0\,\right],\ 1\right\}\cdot\bar{k}$$

The misallocation of resources

The output in the economy is given by

$$\begin{aligned}
Y &= A\cdot k^* + [\bar{k}-k^*] \\
&= AL(\bar{k}) + [\bar{k} - L(\bar{k})] \\
&= A\ \underbrace{\left[(1-\psi) + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right) - \frac{\theta_U^2}{c_{11}}\cdot\theta_C\right]\cdot\theta_C\cdot\max\left[\,\zeta-(1-\psi)\,,\,0\,\right]\right]}_{<1}\cdot\bar{k} \\
&\quad + \left[1 - \underbrace{\left[(1-\psi) + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right) - \frac{\theta_U^2}{c_{11}}\cdot\theta_C\right]\cdot\theta_C\cdot\max\left[\,\zeta-(1-\psi)\,,\,0\,\right]\right]}_{<1}\right]\cdot\bar{k} \\
&= [A\cdot(1-m) + 1\cdot m]\cdot\bar{k} \\
&= [1\cdot \underbrace{(1-m)}_{\text{Fraction of } \bar{k} \text{ invested in productive technology}} + \frac{1}{A}\cdot \underbrace{m}_{\text{Fraction of } \bar{k} \text{ invested in unproductive technology}}]\cdot A\cdot\bar{k}
\end{aligned}$$

and (2.4.14) follows.

1.C.3 Normative Analysis

The problem of a social planner which chooses the optimal level of creditor rights by taking as given the bargaining power of workers

$$\begin{aligned}
\max_{\theta_C\in[0,1]}(1-m(\theta_U,\theta_C,\zeta,\psi)) &= (1-\psi) + (1-\theta_U)^2\cdot\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C - \frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right]\cdot\max\left[\,\zeta-(1-\psi)\,,\,0\,\right] \\
&= (1-\psi) + (1-\theta_U)^2\cdot\max\left[\,\zeta-(1-\psi)\,,\,0\,\right]\cdot\max_{\theta_C\in[0,1]}\left[\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C - \frac{\theta_U^2}{c_{11}}\cdot\theta_C^2\right] \\
&= \max_{\theta_C\in[0,1]}\left(1+\frac{\theta_U^2}{c_{11}}\right)\cdot\theta_C - \frac{\theta_U^2}{c_{11}}\cdot\theta_C^2 \qquad (1.C.7)
\end{aligned}$$

is equivalent to (2.4.15). Hence taking FOC

$$\left(1+\frac{\theta_U^2}{c_{11}}\right) - 2\cdot\frac{\theta_U^2}{c_{11}}\cdot\theta_C = 0$$

$$\theta_C = \frac{1}{2}\cdot\left[\frac{c_{11}}{\theta_U^2}+1\right]$$

we get (2.4.16).

1.D Appendix: Dynamic Model

1.D.1 Proof of Theorem 2.5.1, 2.5.4, 2.5.6

Existence of unique continuous function

Without loss of generality, let us express the nash bargaining problems (2.5.23), (2.5.34), and (2.5.38) as

$$
\begin{aligned}
(Tf)(\underline{p}, \underline{s}) &= \arg\max_{c \in \mathrm{C}} \left\{ A\left(\underline{p}, \underline{s}, c\right)^{(1-\theta)} \cdot B\left(\underline{p}, \underline{s}, c\right)^{\theta} \right\} \\
\text{s.t. } & A\left(\underline{p}, \underline{s}, c\right) \geq 0, \quad B\left(\underline{p}, \underline{s}, c\right) \geq 0
\end{aligned} \tag{1.D.8}
$$

where $c \in \mathrm{C}$ reads $v \in \mathrm{W}$ in (2.5.23), and (2.5.38), and reads $a \in [0,1]$ in (2.5.34). $A(\cdot), B(\cdot)$ are continuous; I will be more precise about their functional forms when needed.

Proof. The proof proceeds in 3 steps.

1. For any $f \in \mathscr{C}^{\mathrm{C}}(\underline{P} \times \underline{S})$ and $(\underline{p}, \underline{s}) \in \underline{P} \times \underline{S}$, $(Tf)(\underline{p}, \underline{s}) \subset \mathbb{R}$ is i) not-empty, ii) compact valued, iii) upperhemicontinuous and $(Tf)(\underline{p}, \underline{s}) : \mathrm{C} \to \mathrm{C}$.

   Proof. Since C is a not-empty, compact valued, continuous feasible correspondence, and the objective function is continuous (product of continuous functions), then by direct application of the Berge's Maximum Theorem the optimal correspondence is not-empty compact-valued, uhc and is contained in the feasible correspondence C. Noticing that $\underline{s}$ was arbitrary the result follows. ☐

2. For any $f \in \mathscr{C}^{\mathrm{C}}(\underline{P} \times \underline{S})$, the product correspondence

   $$(Tf)(\underline{p}, \underline{s}) = \Pi_{(\underline{p}, \underline{s}) \in \underline{P} \times \underline{S}} (Tf)(\underline{p}, \underline{s}) \subset \mathrm{C}$$

   is not empty, compact valued, uhc and $(Tf)(\underline{p}, \underline{s}) : \mathrm{C} \to \mathrm{C}$.

   Proof. The result follows from the fact that: 1) by point 1, $(Tf)(\underline{p}, \underline{s})$ is not empty, compact valued, uhc, included in C; 2) the product correspondence preserves these properties (Aliprantis and Border, 1999: Thm 16.28). ☐

3. Since $\theta \in \Theta \subset [0,1]$, then $(Tf)(\underline{p}, \underline{s})$ is a not-empty, compact and convex-valued uhc correspondence with $(Tf)(\underline{p}, \underline{s}) : \mathrm{C} \to \mathrm{C}$.

   Proof. Lemma (1.D.1) shows that the solution is unique. In particular if $\theta \in \Theta \subset [0,1]$, by strict concavity of the objective function over $[0,1]$, the Nash Bargaining Problem is well defined with a unique continuous solution.
   This implies that the product correspondence $(Tf)(\underline{p})$ is a single-valued, continuous function. ☐

Hence by Kakutani-Fan-Glicksberg FPT there exists a continuous $f^*(\underline{p}, \underline{s}) \in \mathrm{C}$ such that $f(\underline{p}, \underline{s})^* \in (Tf)(\underline{p}, \underline{s})$. Because of Lemma 1.D.1 we know also that the solution is unique (which completes the proof). ☐

Uniqueness

Lemma 1.D.1. *If $\theta \in \Theta \subset [0,1]$, for a given $\underline{p}$ there exists a* ***unique*** *$f(\underline{p}, \underline{s}) \in \mathscr{C}^{\mathrm{C}}(\underline{P} \times \underline{S})$ which solves the Nash Bargaining Problem.*

Proof. Since the proof require to specify the functional forms of $A(\cdot), B(\cdot)$ I will proceed theorem-wise. I start first with the restructuring problems (2.5.34) and (2.5.38), which are differentiable on the whole support, and move eventually to the continuation problem (2.5.23), which is differentiable almost everywhere, except in $d = 0$.
For simplicity, let $h(\cdot) = A(\cdot)^{\theta} B(\cdot)^{1-\theta}$ denote the objective function.

- *Theorem 2.5.4*: uniqueness of solution to the debt restructuring problem
  Taking derivative with respect to a

$$\frac{\partial h(a)}{\partial a} = \alpha^R \left[-(1-\theta_C)\cdot \iota b(S^F)^{-\theta_C}(S^C)^{\theta_C} + \theta_C(S^F)^{1-\theta_C}(S^C)^{\theta_C-1}b\right]$$

  Taking second derivative:

$$\frac{\partial^2 h(a)}{\partial a^2} = -\alpha^R \Bigg\{ \Bigg[\underbrace{\theta_C(1-\theta_C)\cdot(\iota b)^2\cdot(S^F)^{-\theta_C-1}(S^C)^{\theta_C} + \theta_C(1-\theta_C)\cdot \iota b^2(S^F)^{-\theta_C}(S^C)^{\theta_C-1}}_{+}\Bigg] \Bigg[\underbrace{(1-\theta_C)\theta_C(S^F)^{-\theta_C}(S^C)^{\theta_C-1}\iota b^2 + (1-\theta_C)\theta_C(S^F)^{1-\theta_C}(S^C)^{\theta_C-2}b^2}_{+}\Bigg]\Bigg\} < 0$$

  which completes the proof.

- *Theorem 2.5.6*: uniqueness of solution to the labour restructuring problem
  Taking derivative with respect to w

$$\frac{\partial h(v)}{\partial v} = \alpha^R \left[-(1-\theta_U)\cdot \iota n(S^F)^{-\theta_U}(S^W)^{\theta_U} + \theta_U(S^F)^{1-\theta_U}(S^W)^{\theta_U-1}n\right]$$

  Taking second derivative:

$$\frac{\partial^2 h(v)}{\partial v^2} = -\alpha^R \Bigg\{ \Bigg[\underbrace{\theta_U(1-\theta_U)\cdot(\iota n)^2\cdot(S^F)^{-\theta_U-1}(S^W)^{\theta_U} + \theta_U(1-\theta_U)\cdot \iota n^2(S^F)^{-\theta_U}(S^W)^{\theta_U-1}}_{+}\Bigg] \Bigg[\underbrace{(1-\theta_U)\theta_U\cdot(S^F)^{-\theta_U}(S^W)^{\theta_U-1}\iota n^2 + (1-\theta_U)\theta_U(S^F)^{1-\theta_U}(S^W)^{\theta_U-2}n^2}_{+}\Bigg]\Bigg\} < 0$$

  which completes the proof.

- *Theorem 2.5.1*: uniqueness of solution to the wage bargaining problem when the firm continues.
  Taking derivative with respect to w

$$\frac{\partial h(v)}{\partial v} = -(1-\theta_U)\cdot g'(\cdot)\cdot n(S^F)^{-\theta_U}(S^W)^{\theta_U} + \theta_U(S^F)^{1-\theta_U}(S^W)^{\theta_U-1}n$$

  Taking second derivative:

$$\frac{\partial^2 h(v)}{\partial v^2} = -\Bigg\{ \Bigg[\underbrace{\theta_U(1-\theta_U)\cdot(g'(\cdot)\cdot n)^2\cdot(S^F)^{-\theta_U-1}(S^W)^{\theta_U} + \theta_U(1-\theta_U)\cdot g'(\cdot)\cdot n^2(S^F)^{-\theta_U}(S^W)^{\theta_U-1}}_{+}\Bigg] \Bigg[\underbrace{(1-\theta_U)\theta_U\cdot(S^F)^{-\theta_U}(S^W)^{\theta_U-1}g'(\cdot)\cdot n^2 + (1-\theta_U)\theta_U(S^F)^{1-\theta_U}(S^W)^{\theta_U-2}n^2}_{+}\Bigg]\Bigg\} < 0$$

  Since in an interior solution with $d \neq 0$ $g(\cdot) = 1$ if $d > 0$ and $g(\cdot) = \iota$ if $d < 0$, the result follows.

□

1.D.2 The Continuation Problem

Since firm's preferences are monotonic in d, the budget constraint in (2.5.22) is binding, the firm's ordinary choices reduce to (n, b'), and the continuation problem simplifies to

$$V^C(\underline{p},\underline{s}) = \max_{(n,b')\in \mathrm{N}\times\mathrm{B}} g\left(y(\underline{p},x,n) - w(\underline{p},\underline{s},n)\cdot n - \chi_o - \delta k + q(\underline{p},x,b')b' - b\right) + \beta\cdot\mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right]$$
$$\text{s.t.}\quad (\underline{p},\ \underline{s},\ n,\ w(\underline{p},\underline{s},n)) \in W(\underline{p},\underline{s},n)$$

Since labour is not a state variable, the *static* size choice, n - taken in order to maximize profits - does not alter the *inter-temporal* debt choice, b' - taken to smooth dividends over time. Mathematically, since the two controls enter additively in the objective function and the derivative is a linear operator, the problems are separable.
Hence, (2.5.22) becomes

$$V^C(\underline{p},\underline{s}) = \max_{n\in\mathrm{N}}\max_{b'\in\mathrm{B}} g\left(y(\underline{p},x,n) - w(\underline{p},\underline{s},n)\cdot n - \chi_o - \delta k + q(\underline{p},x,b')b' - b\right) + \beta\cdot\mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right]$$
$$\text{s.t.}\quad (\underline{p},\ \underline{s},\ n,\ w(\underline{p},\underline{s},n)) \in W(\underline{p},\underline{s},n)$$

To simplify notation, let the operating profits net of investment and gross of the debt issuance be

$$A(\underline{p},\underline{s},n,b') \equiv y(\underline{p},x,n) - \chi_o + q(\underline{p},x,b')b' - \delta k$$

let the discounted markov operator be

$$E(\underline{p},x,b') \equiv \beta\cdot\mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right]$$

and let omit the dependence of the bargaining power on the region, $\theta_U \equiv \theta_U(r)$.

Proposition 2.5.2 and 2.5.3

By and large, for a given n, we can rewrite (2.5.23)

$$w(\underline{p},\underline{s},n) \equiv \arg\max_{v\in\mathrm{W}} \left[\underbrace{\max_{b'\in\mathrm{B}} g\left[A(\underline{p},\underline{s},n,b') - b - v\cdot n\right] + E(\underline{p},x,b')}_{V^C_{v,n}(\underline{p},\underline{s})}\right]^{(1-\theta_U)} \cdot \left[v\cdot n - \underline{w}\cdot n\right]^{\theta_U}$$
$$\text{s.t. } V^C_{v,n}(\underline{p},\underline{s}) \geq 0, \qquad v \geq \underline{w}$$

Fix b'. By taking first order condition with respect to v

$$(1-\theta_U)\frac{g'(\cdot)}{g\left(A(\underline{p},\underline{s},n,b') - b - vn\right) + E(\underline{p},x,b')}n = \theta_U\frac{1}{v-\underline{w}}$$
$$(1-\theta_U)g'(\cdot)[v-\underline{w}]n = \theta_U\left[g\left(A(\underline{p},\underline{s},n,b') - b - vn\right) + E(\underline{p},x,b')\right] \tag{1.D.9}$$

By definition of $g(d) = \left[\mathbb{I}_{\{d\geq 0\}} + \iota\cdot\mathbb{I}_{\{d<0\}}\right]\cdot d$, we have $g'[\cdot] = 1$ if $d > 0$, $g'[\cdot] = \iota$ if $d < 0$, and $g'[0]$ is not defined. Accordingly, Problem (2.5.23) might have an interior or corner solution.
An interior solution is the wage $w^{interior}(\underline{p},\underline{s},n)$ which satisfies (1.D.9), for either $d > 0$ or $d < 0$.
The existence of an interior solution proceeds by guess and verify: first, I guess that $d > 0$, substitute it in (1.D.9) and check if for $w_{d>0}(\underline{p},\underline{s})$ satisfies it. If not I proceed with $d < 0$.

Let us guess $d > 0$. Than solving (1.D.9) when $g'[\cdot] = 1$, we get

$$(1-\theta_U)g'(\cdot)[v - \underline{w}]n = \theta_U \left[g\left(A(\underline{p}, \underline{s}, n, b') - b - vn\right) + E(\underline{p}, x, b')\right]$$
$$vn = +\underline{w}n + \theta_U \left[A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + E(\underline{p}, x, b')\right]$$

and therefore

$$w^{interior}_{d>0}(\underline{p}, \underline{s}, n)n = \underline{w}n + \theta_U \left[A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + E(\underline{p}, x, b')\right]$$

Then I verify that:

$$d = A(\underline{p}, \underline{s}, n, b') - b - w(\underline{p}, \underline{s}, n)n \geq 0$$

If not, I guess $d < 0$. Than $g'[\cdot] = \iota$ and

$$(1-\theta_U)g'(\cdot)[v - \underline{w}]n = \theta_U \left[g(A(\underline{p}, \underline{s}, n, b') - b - vn) + E(\underline{p}, x, b')\right]$$
$$vn = +\underline{w}n + \theta_U \left[A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + \frac{1}{\iota} \cdot E(\underline{p}, x, b')\right]$$

and therefore

$$w^{interior}_{d<0}(\underline{p}, \underline{s}, n)n = \underline{w}n + \theta_U \left[A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + \frac{1}{\iota} \cdot E(\underline{p}, x, b')\right]$$

Then I verify that

$$d = A(\underline{p}, \underline{s}, n, b') - b - w(\underline{p}, \underline{s}, n)n < 0$$

Let us denote the nash bargaining surplus of the firm

$$S(\underline{p}, \underline{s}, n) = \begin{cases} \max_{b' \in \mathrm{B}} A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + E(\underline{p}, x, b') & d > 0 \\ \max_{b' \in \mathrm{B}} A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + \frac{1}{\iota} \cdot E(\underline{p}, x, b') & d < 0 \end{cases}$$

or more compactly

$$S(\underline{p}, \underline{s}, n) \equiv \max_{b' \in \mathrm{B}} A(\underline{p}, \underline{s}, n, b') - b - \underline{w}n + \beta \cdot \frac{1}{\mathbb{I}_{\{d \geq 0\}} + \iota \cdot \mathbb{I}_{\{d<0\}}} + E(\underline{p}, x, b')$$

Then we can rewrite the interior solutions

$$w^{interior}(\underline{p}, \underline{s}, n)n = \underline{w}n + \theta_U S(\underline{p}, \underline{s}, n)$$

and the value of continuation for a given number of workers

$$V^C_n(\underline{p}, \underline{s}) = (1-\theta_U)S(\underline{p}, \underline{s}, n)$$

Since the b' which maximizes $V^C_n(\underline{p}, \underline{s})$ coincides with the b' which maximizes the nash bargaining surplus $S(\underline{p}, \underline{s}, n)$, then it maximizes both $w^{interior}_{d<0}$, $w^{interior}_{d>0}$. Equations (2.5.24), (2.5.25) follows. The firm chooses n to maximize

$$\max_{n \in \mathrm{N}} (1-\theta_U) \cdot S(\underline{p}, \underline{s}, n)$$

and (2.5.26) follow.
Let $b'^{,*}$ be the optimal choice of debt, manipulating and substituting

$$(1-\theta_U) \max_{n \in \mathrm{N}} A(\underline{p}, \underline{s}, n, b'^{,*}) - b - \underline{w}n + \frac{1}{\mathbb{I}_{\{d \geq 0\}} + \iota \cdot \mathbb{I}_{\{d<0\}}} \cdot E(\underline{p}, x, b'^{,*})$$

Since hiring is a static choice, by taking FOC and using (3.3.1):

$$\alpha\eta(z \cdot x)^{(1-\alpha\eta)} k^{(1-\alpha)\eta} n^{\alpha\eta - 1} = \underline{w}$$

$$n = z \cdot x \left(\frac{\alpha\eta k^{(1-\alpha)\eta}}{\underline{w}} \right)^{\frac{1}{1-\alpha\eta}}$$

Equations (2.5.27) follows.
Hence substituting in (3.3.1)

$$\begin{aligned} y(\underline{p}, x, n^*) &= (z \cdot x)^{(1-\alpha\eta)} (k^{1-\alpha} n^{*,\alpha})^{\eta} \\ &= (z \cdot x) \left(\frac{\alpha\eta}{\underline{w}} \right)^{\frac{\alpha\eta}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}} \end{aligned}$$

and equation (2.5.28) follows.

Let me now turn to the corner solutions, i.e. the wage for which $d = 0$

$$w^{corner}(\underline{p}, \underline{s}, n) = \frac{A(\underline{p}, \underline{s}, n, b') - b}{n}$$

Given n, a solution to (2.5.23) is the wage which maximizes the nash-bargaining product.

$$\max\{V^C_{w^{interior},n}(\underline{p}, \underline{s})^{(1-\theta_U(r))} \cdot [w^{interior} \cdot n - \underline{w} \cdot n]^{\theta_U(r)}, \max\{V^C_{w^{corner},n}(\underline{p}, \underline{s})^{(1-\theta_U(r))} \cdot [w^{corner} \cdot n - \underline{w} \cdot n]^{\theta_U(r)}\}$$

When $w^{corner}(\underline{p}, \underline{s}, n)$ solves (2.5.23), there is indeterminacy of n. To see this

$$\begin{aligned} V^C_{w^{corner},n}(\underline{p}, \underline{s}) &= \max_{b' \in \mathrm{B}} g \left[A(\underline{p}, \underline{s}, n, b'^{,*}) - b - \underline{w}^{corner} n \right] + E(\underline{p}, x, b') \\ &= \max_{b' \in \mathrm{B}} g \left[A(\underline{p}, \underline{s}, n, b'^{,*}) - b - (A(\underline{p}, \underline{s}, n, b'^{,*}) - b) \right] + E(\underline{p}, x, b') \end{aligned}$$

Hence only $w^{corner} \cdot n$ is determined. This is a computationally interesting case. How do I deal with it? Since $\lim_{d \to 0^+} n^*(\underline{p}, \underline{s}) = \lim_{d \to 0^-} n^*(\underline{p}, \underline{s}) = n^*(\underline{p}, \underline{s}) = z \cdot x \cdot \left(\frac{\alpha\eta}{\underline{w}} \right)^{\frac{1}{1-\alpha\eta}} k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}$ to preserve continuity, I assume that $n^*_{d=0} = n^*$ as well. In words, since a firm $(\underline{p}, \underline{s})$ which distribute a small amount of dividends chooses the same amount of worker $n^*(\underline{p}, \underline{s})$, as if it were issuing a small amount of equity, than I assume it makes the same hiring choice when it does not distribute dividends. I do not have a counter-argument why the continuity should not hold, i.e. what is the rationale why a firm that distribute [issue] a small amount of dividends [equity] differ dramatically in its hiring choices than the same firm that does not distribute dividends.

1.D.3 The Reorganization Problem

Since firm's preferences are monotonic in d, the budget constraint in (2.5.30) is binding, the firm's ordinary choices reduce to (n, b'), and the reorganization problem simplifies to

$$\begin{aligned} V^R(\underline{p}, \underline{s}) = \max_{e \in \mathrm{E}} \ & \alpha^R(e; \theta_U(r)) \cdot \\ & \left[\max_{(n,b') \in \mathrm{N} \times \mathrm{B}_+} \iota \left[y(\underline{p}, x, n) - w^R(\underline{p}, \underline{s}, e, n) \cdot n - \chi_o - \delta k + q(\underline{p}, x, b') \cdot b' - \alpha^C(\underline{p}, \underline{s}, e, n, w^R(\underline{p}, \underline{s}, e, n)) \cdot b \right] + \beta \cdot \mathbb{E}_{x'|x} \left[V(\underline{p}, \underline{s}') \right] \right] - c(e) \\ \text{s.t.} \quad & y(\underline{p}, x, n) - w^R(\underline{p}, \underline{s}, e, n) \cdot n - \chi_o - \delta k + q(\underline{p}, x, b') \cdot b' - \alpha^C(\underline{p}, \underline{s}, e, n, w^R(\underline{p}, \underline{s}, e, n)) \cdot b < 0 \qquad (\textit{Equity Issuance}) \\ & (\underline{p}, \underline{s}, n, v, e, \alpha^C(\underline{p}, \underline{s}, e, n, v)) \in A^C(\underline{p}, \underline{s}, e, n, v) \\ & (\underline{p}, \underline{s}, n, e, w^R(\underline{p}, \underline{s}, e, n)) \in W^R(\underline{p}, \underline{s}, e, n) \end{aligned}$$

Since labour is not a state variable, the *static* size choice, n - taken in order to maximize profits - does not alter the *inter-temporal* debt choice, b' - taken to smooth dividends over time. Mathematically, since the two controls enter additively in the objective function and the derivative is a linear operator, the problems are separable.
Hence, (2.5.22) becomes

$$\begin{aligned} V^R(\underline{p},\underline{s}) = \max_{e\in \mathrm{E}} \; & \alpha^R(e;\theta_U(r))\cdot \\ & \left[\max_{n\in \mathrm{N}} \max_{b'\in \mathrm{B}_+} \iota \left[y(\underline{p},x,n) - w^R(\underline{p},\underline{s},e,n)\cdot n - \chi_o - \delta k + q(\underline{p},x,b')\cdot b' - \alpha^C(\underline{p},\underline{s},e,n,w^R(\underline{p},\underline{s},e,n))\cdot b\right] + \beta \cdot \mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right]\right] - c(e) \\ \text{s.t.} \quad & y(\underline{p},x,n) - w^R(\underline{p},\underline{s},e,n)\cdot n - \chi_o - \delta k + q(\underline{p},x,b')\cdot b' - \alpha^C(\underline{p},\underline{s},e,n,w^R(\underline{p},\underline{s},e,n))\cdot b < 0 \qquad (\textit{Equity Issuance}) \\ & (\underline{p},\underline{s},n,v,e,\alpha^C(\underline{p},\underline{s},e,n,v)) \in A^C(\underline{p},\underline{s},e,n,v) \\ & (\underline{p},\underline{s},n,e,w^R(\underline{p},\underline{s},e,n)) \in W^R(\underline{p},\underline{s},e,n) \end{aligned}$$

To simplify notation, let the operating profits net of investment and gross of the debt issuance be

$$A(\underline{p},\underline{s},n,b') \equiv y(\underline{p},x,n) - \chi_o + q(\underline{p},x,b')b' - \delta k$$

let the discounted markov operator be

$$E(\underline{p},x,b') \equiv \beta \cdot \mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}')\right]$$

and let omit the dependence of the bargaining power on the region, $\theta_U \equiv \theta_U(r)$.
To simplify notation, let

$$A(\underline{p},\underline{s},n,b') = y(\underline{p},x,n) - \chi_o + q(\underline{p},x,b'^{,*}_{e,n,v}(\underline{p},\underline{s}))b'^{,*}_{e,n,v}(\underline{p},\underline{s}) - \delta k$$

and let

$$E(\underline{p},\underline{s}) = \beta \cdot \mathbb{E}_{x'|x}\left[V(\underline{p},\underline{s}'))\right]$$

Proposition 2.5.5

By and large, for a given (e,n,v) we can rewrite (2.5.34)

$$\alpha^C(\underline{p},\underline{s},e,n,v) \equiv \operatorname*{arg\,max}_{a\in[0,1]} \left\{ \underbrace{[S^F_{e,n,v}\left(\underline{p},\underline{s};a\right)]}_{\text{Surplus Firm}}{}^{(1-\theta_C)} \cdot \underbrace{[S^C_{e,n,v}\left(\underline{p},\underline{s};a\right)]}_{\text{Surplus Creditors}}{}^{\theta_C} \right\}$$

$$\text{s.t. } S^F_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0, \quad S^C_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0$$

Hence using the definition of firm surplus (2.5.32) and credit intermediary surplus (2.5.33), we rewrite (2.5.34)

$$\begin{aligned} &\alpha^C(\underline{p},\underline{s},e,n,v) = \operatorname*{arg\,max}_{a\in \mathrm{A}} \left[\alpha^R \cdot [\max_{b'\in \mathrm{B}_+} \iota \cdot [A(\underline{p},\underline{s},n,b') - vn] + E(\underline{p},\underline{s}) - \iota ab] \right]^{(1-\theta_C)} \cdot \left[\alpha^R ab + (1-\alpha^R)R^7(\underline{p},\underline{s}) - R^7(\underline{p},\underline{s})\right]^{\theta_C} \\ &\text{s.t. } S^F_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0, \quad S^C_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0, \quad d \leq 0, \end{aligned}$$

where $\alpha^R(e;\theta_U(r)) = \alpha^R$. By simplifying it further,

$$\begin{aligned} &\alpha^C(\underline{p},\underline{s},e,n,v) = \alpha^R(e;\theta_U(r)) \cdot \operatorname*{arg\,max}_{a\in \mathrm{A}} \left[\max_{b'\in \mathrm{B}_+} \iota \cdot [A(\underline{p},\underline{s},n,b') - vn] + E(\underline{p},\underline{s}) - \iota ab \right]^{(1-\theta_C)} \cdot \left[ab - R^7(\underline{p},\underline{s})\right]^{\theta_C} \\ &\text{s.t. } S^F_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0, \quad S^C_{e,n,v}\left(\underline{p},\underline{s};a\right) \geq 0, \quad d \leq 0, \end{aligned}$$

Since $a \in A \equiv [0,1]$ and $b \in \mathrm{B}$, I make the following change of variable $r = ab \in [0, b_{\max}] \subseteq R_{+}$ and I get the equivalent representation

$$\alpha^C(\underline{p}, \underline{s}, e, n, v) = \alpha^R(e; \theta_U(r)) \cdot \underset{r\in[0,b_{\max}]}{\arg\max} \left[\max\left\{ \max_{b'\in \mathrm{B}_+} \iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r, 0 \right\} \right]^{(1-\theta_C)} \cdot \left[\max\{r - R^7(\underline{p}, \underline{s}), 0\} \right]^{\theta_C}$$

Since the debt is chosen over a discrete finite set, $b' \in \mathrm{B} \equiv \{b_{\min}, \ldots, b_{\max}\} \subset \mathbb{R}$, then we can solve the problem for any b'

$$\alpha^C(\underline{p}, \underline{s}, e, n, v; b') = \alpha^R(e; \theta_U(r)) \cdot \underset{r\in[0,b_{\max}]}{\arg\max} \left[\max[\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r, 0] \right]^{(1-\theta_C)} \cdot \left[\max[r - R^7(\underline{p}, \underline{s}), 0] \right]^{\theta_C}$$

and then choose the optimal level of debt such that

$$b^{',*} = \underset{b'\in \mathrm{B}_+}{\arg\max}\, \alpha^C(\underline{p}, \underline{s}, e, n, v; b')$$

Then for a given b', by taking first order conditions

$$(1-\theta_C) \cdot \frac{\iota}{\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r} = \theta_C \cdot \frac{1}{r - R^7(\underline{p}, \underline{s})}$$

$$r = R^7(\underline{p}, \underline{s}) + \theta_C \cdot \left[A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - R^7(\underline{p}, \underline{s}) \right]$$

Let the nash bargaining surplus in debt restructuring for a given (e, n, v) be

$$S^R_{n,v,e}(\underline{p}, \underline{s}) = \max\{A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - R^7(\underline{p}, \underline{s}), 0\}$$

Clearly, for an interior solution to exist the nash bargaining surplus has to be (strictly) greater than zero $S^R_{n,v,e}(\underline{p}, \underline{s}) > 0$. Then upon success, the recovery value under Ch 11 is

$$r^* = R^7(\underline{p}, \underline{s}) + \theta_C \cdot \max\left\{ A(\underline{p}, \underline{s}, n, b') - vn - R^7(\underline{p}, \underline{s}) + \frac{1}{\iota} E(\underline{p}, \underline{s}), 0 \right\}$$

Lemma 1.D.2. *The optimal level of debt in possession financing $b^{',*}$ and the Ch 11 recovery value upon success r^* do not depend on the level of effort exerted e.*

Proof. The result comes by noticing that maximizing

$$\alpha^C(\underline{p}, \underline{s}, e, n, v) = \alpha^R(e; \theta_U(r)) \cdot \underset{r\in[0,b_{\max}]}{\arg\max} \left[\max[\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r, 0] \right]^{(1-\theta_C)} \cdot \left[\max[r - R^7(\underline{p}, \underline{s}), 0] \right]^{\theta_C}$$

tantamounts to maximize

$$\alpha^C(\underline{p}, \underline{s}, e, n, v) = \underset{r\in[0,b_{\max}]}{\arg\max} \left[\max[\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r, 0] \right]^{(1-\theta_C)} \cdot \left[\max[r - R^7(\underline{p}, \underline{s}), 0] \right]^{\theta_C}$$

□

Hence we can drop the dependence of the nash bargaining surplus on e, $S^R_{n,v}(\underline{p}, \underline{s})$, and considering the optimal debt $b^{',*}$ equation (2.5.35) follows.

Substituting r^* in (2.5.32) we get the expected reorganization value of a firm after debt restructuring for a given (e, n, v)

$$
\begin{aligned}
&S^F_{e,n,v}\left(\underline{p}, \underline{s}\right) \\
&= \alpha^R(e; \theta_U(r)) \cdot \left[\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn] + E(\underline{p}, \underline{s}) - \iota r^*\right] \\
&= \alpha^R(e; \theta_U(r)) \cdot \iota \cdot \left[A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - r^*\right] \\
&= \alpha^R(e; \theta_U(r)) \cdot \iota \cdot \left[A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - R^7(\underline{p}, \underline{s}) - \theta_C \cdot \max\left\{A(\underline{p}, \underline{s}, n, b') - vn - R^7(\underline{p}, \underline{s}) + \frac{1}{\iota} E(\underline{p}, \underline{s}), 0\right\}\right] \\
&= \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot \iota \cdot \max\left\{A(\underline{p}, \underline{s}, n, b') + \frac{1}{\iota} E(\underline{p}, \underline{s}) - vn - R^7(\underline{p}, \underline{s}), 0\right\}
\end{aligned}
$$

and equation (2.5.37) follows

$$
S^F_{e,n,v}(\underline{p}, \underline{s}) = \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot \iota \cdot S^R_{n,v}(\underline{p}, \underline{s})
$$

Similarly, substituting r^* we get the expected recovery value under Ch 11

$$
\begin{aligned}
R^{11}_{e,n,v}(\underline{p}, \underline{s}) &\equiv \alpha^R(e; \theta_U(r)) r^* + (1 - \alpha^R(e; \theta_U(r))) R^7(\underline{p}, \underline{s}) \\
&= \alpha^R(e; \theta_U(r)) \left[R^7(\underline{p}, \underline{s}) + \theta_C \cdot \max\left\{A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - R^7(\underline{p}, \underline{s}), 0\right\}\right] + (1 - \alpha^R(e; \theta_U(r))) R^7(\underline{p}, \underline{s}) \\
&= R^7(\underline{p}, \underline{s}) + \alpha^R(e; \theta_U(r)) \cdot \theta_C \cdot \max\left\{A(\underline{p}, \underline{s}, n, b') - vn + \frac{1}{\iota} E(\underline{p}, \underline{s}) - R^7(\underline{p}, \underline{s}), 0\right\}
\end{aligned}
$$

and (2.5.36) follows.

Proposition 2.5.7

Let us report the labour restructuring problem (2.5.38)

$$
\begin{aligned}
&(W^R w)(\underline{p}, \underline{s}, e, n) = \underset{v \in \mathrm{W}}{\arg\max}\ [S^F_{e,n,v}(\underline{p}, \underline{s})]^{(1-\theta_U)} \ \cdot \ [\alpha^R(e; \theta_U(r)) \cdot [v \cdot n - \underline{w} \cdot n]]^{\theta_U} \\
&\text{s.t. } S^F_{e,n,v}(\underline{p}, \underline{s}) \geq 0, \qquad v \geq \underline{w}
\end{aligned}
$$

Substituting (2.5.37)

$$
\begin{aligned}
&(W^R w)(\underline{p}, \underline{s}, e, n) = \underset{v \in \mathrm{W}}{\arg\max}\ [\alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C) \cdot S^R_{n,v}(\underline{p}, \underline{s})]^{(1-\theta_U)} \ \cdot \ [\alpha^R(e; \theta_U(r)) \cdot [v \cdot n - \underline{w} \cdot n]]^{\theta_U} \\
&\text{s.t. } S^F_{e,n,v}(\underline{p}, \underline{s}) \geq 0, \qquad v \geq \underline{w}
\end{aligned}
$$

and simplifying

$$
\begin{aligned}
&(W^R w)(\underline{p}, \underline{s}, e, n) \\
&= \alpha^R(e; \theta_U(r)) \cdot (1 - \theta_C)^{(1-\theta_U)} \cdot \underset{v \in \mathrm{W}}{\arg\max}\ [\max\left\{\iota \cdot [A(\underline{p}, \underline{s}, n, b') - vn - R^7(\underline{p}, \underline{s})] + E(\underline{p}, \underline{s}), 0\right\}]^{(1-\theta_U)} \ \cdot \ [v \cdot n - \underline{w} \cdot n]^{\theta_U} \\
&\text{s.t. } S^F_{e,n,v}(\underline{p}, \underline{s}) \geq 0, \qquad v \geq \underline{w}
\end{aligned}
$$

By taking first order conditions,

$$(1-\theta_U)\cdot\frac{n\cdot\iota}{\iota\cdot[A(\underline{p},\underline{s},n,b')-vn-R^7(\underline{p},\underline{s})]+E(\underline{p},\underline{s})}=\theta_U\cdot\frac{n}{v\cdot n-\underline{w}\cdot n}$$

$$v\cdot n=\underline{w}\cdot n+\theta_U\cdot\left[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}\cdot n+\frac{1}{\iota}E(\underline{p},\underline{s})\right]$$

Let the labour restructuring nash bargaining surplus

$$S_n^R(\underline{p},\underline{s})=\max\{A(\underline{p},\underline{s},n,b')+\frac{1}{\iota}E(\underline{p},\underline{s})-R^7(\underline{p},\underline{s})-\underline{w}\cdot n,0\}$$

Clearly, for an interior solution to exist the nash bargaining surplus has to be (strictly) greater than zero $S_n^R(\underline{p},\underline{s})>0$. Equation (2.5.39) follows.
Then we get the wage compensation which maximizes the labour restructuring problem

$$w(\underline{p},\underline{s},e,n)=\underline{w}+\theta_U\cdot\frac{\left[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}\cdot n+\frac{1}{\iota}E(\underline{p},\underline{s})\right]}{n}$$

as in equation (2.5.40), and the expected surplus of the workers is

$$S_n^W=\alpha^R(e;\theta_U(r))\cdot\theta_U\cdot S_n^R(\underline{p},\underline{s})$$

Similarly, substituting in (2.5.37) we get the expected reorganization value of a firm after restructuring the labour

$$\begin{aligned}
&S_{e,n}^F\left(\underline{p},\underline{s}\right)\\
&=\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot\max\left\{\iota\cdot[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-w(\underline{p},\underline{s},e,n)n]+E(\underline{p},\underline{s}),0\right\}\\
&=\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot\max\left\{\iota\cdot[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}n-\theta_U\cdot\left[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}\cdot n+\frac{1}{\iota}E(\underline{p},\underline{s})\right]]+E(\underline{p},\underline{s}),0\right\}\\
&=\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot\max\left\{\iota\cdot\left[(1-\theta_U)\cdot[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}n]-\theta_U\frac{1}{\iota}E(\underline{p},\underline{s})\right]+E(\underline{p},\underline{s}),0\right\}\\
&=\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot(1-\theta_U)\cdot\iota\cdot\max\left\{[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}n+\frac{1}{\iota}\cdot E(\underline{p},\underline{s})],0\right\}
\end{aligned}$$

from which equation (2.5.42) follows

$$S_{e,n}^F\left(\underline{p},\underline{s}\right)=\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot(1-\theta_U)\cdot\iota\cdot S_n^R(\underline{p},\underline{s})$$

Similarly, substituting in (2.5.36) we get the expected recovery value under Ch 11

$$\begin{aligned}
&R_{e,n,v}^{11}(\underline{p},\underline{s})\\
&=R^7(\underline{p},\underline{s})+\alpha^R(e;\theta_U(r))\cdot\theta_C\cdot\max\left\{A(\underline{p},\underline{s},n,b')-w(\underline{p},\underline{s},e,n)n+\frac{1}{\iota}E(\underline{p},\underline{s})-R^7(\underline{p},\underline{s}),0\right\}\\
&=R^7(\underline{p},\underline{s})+\alpha^R(e;\theta_U(r))\cdot\theta_C\cdot\max\left\{A(\underline{p},\underline{s},n,b')+\frac{1}{\iota}E(\underline{p},\underline{s})-R^7(\underline{p},\underline{s})-\left[\underline{w}n+\theta_U\cdot\left[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}\cdot n+\frac{1}{\iota}E(\underline{p},\underline{s})\right]\right],0\right\}\\
&=R^7(\underline{p},\underline{s})+\alpha^R(e;\theta_U(r))\cdot\theta_C\cdot(1-\theta_U(r))\cdot\max\left\{A(\underline{p},\underline{s},n,b')+\frac{1}{\iota}E(\underline{p},\underline{s})-R^7(\underline{p},\underline{s})-\underline{w}n,0\right\}
\end{aligned}$$

and equation (2.5.41) follows

$$R_{e,n}^{11}(\underline{p},\underline{s})=R^7(\underline{p},\underline{s})+\alpha^R(e;\theta_U(r))\cdot\theta_C\cdot(1-\theta_U(r))\cdot S_n^R(\underline{p},\underline{s})$$

In conclusion the expected surplus of the workers

$$
\begin{aligned}
S^W_{e,n}\left(\underline{p},\underline{s}\right) &= \alpha^R(e;\theta_U(r))\left[w(\underline{p},\underline{s},e,n)n-\underline{w}n\right]\\
&= \alpha^R(e;\theta_U(r))\cdot\theta_U\cdot\left[A(\underline{p},\underline{s},n,b')-R^7(\underline{p},\underline{s})-\underline{w}\cdot n+\frac{1}{\iota}E(\underline{p},\underline{s})\right]\\
&= \alpha^R(e;\theta_U(r))\cdot\theta_U\cdot S^R_n(\underline{p},\underline{s}) \qquad (1.D.10)
\end{aligned}
$$

Proposition 2.5.8

The firm chooses n to maximize (2.5.42).

$$\max_{n\in\mathrm{N}}\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot(1-\theta_U(r))\cdot\iota\cdot S^R_n(\underline{p},\underline{s})$$

or equivalently to maximize the nash bargaining surplus

$$\alpha^R(e;\theta_U(r))\cdot(1-\theta_C)\cdot(1-\theta_U(r))\cdot\iota\max_{n\in\mathrm{N}}\cdot S^R_n(\underline{p},\underline{s})$$

which since the hiring choice is a static decision, is equivalent in maximizing

$$\max_{n\in\mathrm{N}}A(\underline{p},\underline{s},n,b')-\underline{w}\cdot n=\max_{n\in\mathrm{N}}y(\underline{p},x,n)-\chi_o+q(\underline{p},x,b')b'-\delta k$$

and simplifying

$$\max_{n\in\mathrm{N}}y(\underline{p},x,n)-\underline{w}\cdot n$$

The firm chooses the number of workers which equates the marginal product of labour to the outside opportunity cost of workers.
Taking FOC and using (3.3.1) equation (2.5.44) follows.

$$n=z\cdot x\left(\frac{\alpha\eta k^{(1-\alpha)\eta}}{\underline{w}}\right)^{\frac{1}{1-\alpha\eta}}$$

Hence substituting in (3.3.1)

$$
\begin{aligned}
y(\underline{p},x,n^*) &= (z\cdot x)^{(1-\alpha\eta)}(k^{1-\alpha}n^{*,\alpha})^\eta\\
&= (z\cdot x)\left(\frac{\alpha\eta}{\underline{w}}\right)^{\frac{\alpha\eta}{1-\alpha\eta}}k^{\frac{(1-\alpha)\eta}{1-\alpha\eta}}
\end{aligned}
$$

and equation (2.5.45) follows. Substituting the solution in (2.5.41), (2.5.42) and (1.D.10), then (2.5.47), (2.5.48) and (2.5.49) follows.

CHAPTER 3

Financial Development, Default Rates and Credit Spreads[1]

3.1 Introduction

We study the joint dynamics of corporate default and credit spreads from 1950 to 2012. We document that, over the last thirty years, defaults rates rose by 467% while credit spreads barely moved. We refer to this evidence as the diverging trend between rising default rates and constant credit spreads.

We provide statistical support for the presence of one structural break in the unconditional mean of default rates around 1984. This date splits the series of default rates in two samples with strikingly different characteristics. On one hand, during the 1950's and 1960's the US economy recorded almost no bankruptcies: the average default rate from 1950 to 1983 equals 0.3%. On the other hand, from the 1980's on we observe a dramatic rise in the number of defaults: the average number of corporate bankruptcies from 1984 to 2012 equals 1.7%. Hence, default rates have increased by 467% throughout the last thirty years. Conversely, the time series of credit spreads does not display any structural shift in its unconditional mean. The average credit spread over the period 1950-1983 records 91 basis points whereas the average spread from 1984 to 2012 amounts to 102 basis points. We run a battery of tests and show that this 11 basis points increase is not statistically significant.

At a first glance, it is hard to reconcile the different behavior of default rates and credit spreads. Anecdotal evidence would suggest the two time series to move together. The credit spread is a market measure of default risk and for this reason it should capture relevant information about default rates[1]. Therefore, such a steep rise in default rates should allegedly be mirrored by credit spreads. However it does not.

To understand this phenomenon, we propose an explanation based on a structural change in the supply side of credit. Although we acknowledge that changes in financial factors, such as shocks to liquidity or to the credit ratings, could account for this diverging trend, we provide a theory that is based just on fundamentals. We conjecture that the reduction in the cost of borrowing due to the widely documented process of deregulation and innovation incurred by the financial sector in the 70s might explain this empirical evidence. Apropos, we construct a dynamic equilibrium model where two features, the development of credit markets and the limited enforceability of debt, can be accounted for the diverging trend between default rates and credit spreads. We model the development of credit markets in a reduced form, as an exogenous reduction of the fixed cost of borrowing. We find that financial development can explain 64% of the observed increase

[1]This Chapter is a joint work with Omar Rachedi.

[1]Longstaff et al. [2005] document that 71% of the Baa yield is explained by default risk.

in average default rates and predict just a 2 basis points increase in the credit spreads. As a robustness check, our explanation quantitatively accounts for a number of trends that have characterized public firms over the last decades: the fall in the number of firms distributing dividends, the rise in the degree of dividend smoothing, and the increase in the idiosyncratic volatility of public firms.

In order to illustrate the model mechanism let us first discuss the implications of the main friction in our economy: limited enforceability of debt contracts. In the model there is a distribution of heterogeneous firms that can default on their debt. In such an event the credit intermediary seizes the assets of the firm. This environment generates endogenous borrowing constraints which depend on the level of capital of the firm, its idiosyncratic efficiency and the demanded amount of debt. In particular, firms with less collateral face tighter constraint because upon default credit intermediaries incur in higher losses. Secondly, less efficient firms face tighter borrowing constraints because they have a higher probability to default in the next period[2]. Finally, larger loans increase the probability of default, by raising the number of scenarios where the firm will not be able to repay its debt. Accordingly, the interest rate which is charged on the loan by the credit intermediary reflects these different determinants of the expected default cost. In conclusion, large or efficient firms can borrow more (or borrow the same quantity at a cheaper price) with respect to ceteris paribus smaller or less efficient ones. In addition to these features, we assume the presence of a fixed borrowing cost that further reduces the financing ability of all firms, hitting disproportionately small firms. What happens with the development of the credit markets? What happens when fixed costs of borrowing are reduced? A reduction in the fixed cost of borrowing has both direct and indirect effects. The *direct effect* is straightforward and twofold. First of all, there is a reduction in credit rationing. Firms can now benefit from the possibility of accessing small amount of loans, before unfeasible because of the presence of a fixed borrowing cost[3]. Secondly, firms can either raise the same amount of debt at a cheaper price or, equivalently, access at the same price a higher amount of loan (just reallocating the resources before devoted to the payment of the borrowing cost to increase the amount of actual loan). The *indirect effect* is the result of the dynamic response of firms to the new environment. To understand it, we need to look more closely at the optimizing behavior of a firm in presence of endogenous borrowing constraints. In our model firms maximize the expected discounted value of the stream of dividends. The presence of endogenously convex loan price schedules makes the value function of the risk-neutral firms to be concave. For this reason, firms seek to smooth dividends against idiosyncratic shocks. Debt is a channel for doing so. Nonetheless the higher the fixed borrowing cost, the tighter the borrowing constraint, and, accordingly, the more difficult for the firm to use efficiently debt for this purpose. In order to partially overcome this obstacle firms try to build up physical capital. Physical capital is in fact the collateral against which firms can borrow at a cheaper price. The result is that firms which have been lucky in experiencing a raw of good productivity shocks tend to accumulate for precautionary reasons more physical capital than what might be motivated just by efficiency

[2]Since the idiosyncratic shock is persistent, firms' actual status predicts their future productivity. If we assume independent idiosyncratic productivity shocks, the borrowing constraint would not depend on the actual efficiency of the firm.

[3]For example, suppose that before the credit market development a firm optimal loan (given the interest rate) was 100$ gross of the borrowing cost, and suppose that the cost of the borrowing process was 200$. The cost of the process is higher than the total amount of the loan required, therefore the firm would have not entered that contract. After financial development, there will be less firms constrained in this fashion.

reasons. Therefore, the higher the level of fixed cost of borrowing, the higher the amount of physical capital devoted for this purpose. Indeed, the collateral value of capital decreases with the fixed cost of borrowing. Conversely, firms which have not been lucky/small firms struggle to optimally exploit profitable investment opportunities, given the high cost of debt. As a conclusion in this economy inefficiently large firms coexist with small firms which struggle to grow.

By reducing the fixed cost of borrowing, financial development significantly affects those dynamics. The reduction of borrowing costs eases firms' access to debt. Efficient small firms can finance more investment and grow, while inefficient firms can reduce their size without being penalized as much as before on their interest rates, due to the lack of collateral. Hence, inefficient large firms shrinks down their scale of operation. As a consequence, given the higher collateral value of capital, firms can borrow more debt for the same amount of capital, implying an increase in leverage. Together with a higher volatility of debt, this implies a higher volatility of leverage. Higher level and volatility of leverage boost the likelihood that firms end up in states of the world where they find optimal to default, pushing up the overall default rate of the economy.

Why the rise in default risk does not translate in an increase of the credit spreads? This question requires a quantitative answer, because the change in credit spreads is driven by two counteracting forces which exert their influence through three channels: the fixed cost of borrowing, the quantity of risk and the loss given default for the credit intermediaries. On the one hand, rising default rates increase the quantity of risk bore by credit intermediaries with the consequence that credit spreads have to rise too. On the other hand, there are two channels through which financial development reduces the credit spreads: the fixed cost of borrowing and the loss given default. First of all, ceteris paribus financial development reduces by construction the fixed cost of borrowing, and therefore the interest rate charged on the loan. The impact on the interest rate is stronger the higher is the expected probability of default of the firm, contributing to the reduction of the credit spread. Secondly, and more importantly from a quantitative point of view, financial development make less stringent the borrowing constraint by allowing firms to operate at a more efficient scale. As a result, the average size and profits increase, implying larger ex-ante liquidation values in case of default. This channel tempers the loss given default for the credit intermediaries, pushing down credit spreads. In the model, financial development makes default to rise from 0.3% to 1.2%. Yet, credit spreads rise just by 2 basis points because the higher default risk is offset by a 24% upsurge in the median expected recovery rate. The bulk of this increase comes from a boost in the profits of the firm, which go up by 21.73%. The median size of capital rises too, by 9.34%.

The model also predicts a number of trend that characterized public firms over the recent decades. First, we show that the reduction of the fixed credit costs changes firms' optimal decisions of dividend payout. After financial development firms are in fact more able to smooth dividends over time, and they can trade off this reduction in volatility with a decrease in the level of dividends. The reduction of the borrowing costs makes the measure of firms distributing dividends to shrink down by 34%. This number accounts for the 73% of the decline documented for the U.S. by Fama and French [2001]. Furthermore, in the model firms also increase the degree of dividend smoothing by a magnitude which is remarkably close to the values estimated by Leary and Michaely [2011] on US public firms. Second, we study the volatility of firms' returns and sales. Indeed, Campbell et al. [2001], Comin and Mulani [2006], Comin and Philippon [2005] show the presence of a secular upward trend in the volatilities of firms. We suggest that this empirical evidence can be (at least partially) accounted for by financial development. Indeed, the model is able to reproduce a rise of 72 % in the volatility of sales and 67% for firms' returns.

3.1.1 Related Literature

This paper adds to the literature on the role of credit markets on firm dynamics. The seminal paper in this field is Cooley and Quadrini [2001], which augments the environment of Hopenhayn [1992] with financial frictions, namely an equity issuance and a bankruptcy deadweight loss. The authors present a model where the dynamics of firms, in terms of growth, job reallocation and exit, is negatively correlated with their initial size and age, as it is in the data. Following Cooley and Quadrini [2001], many papers attempted to understand qualitatively and quantitatively the role of financial frictions on firm characteristics, firm dynamics and the behavior of macroeconomic aggregates. Jermann and Quadrini [2012] show how the limited enforceability of firms' debt might generate endogenous borrowing constraints, which affects not only the dynamics of individual firms, but even the behavior of aggregate financial and real variables. Jermann and Quadrini [2008] use a similar model to show that financial development can be accounted for the rise in volatility of aggregate financial variables and the decline of the volatility of real economic activity. All these models share a common feature: despite firms are allowed to renege on their debt, there is no default in equilibrium. This result stems from the presence of an enforcement constraint, which binds in equilibrium, impeding the firms to default. Recently, few papers have relaxed this condition allowing for equilibrium default. Arellano et al. [2011] build a general equilibrium model which allows for equilibrium default, where financial frictions interact with increases in uncertainty at the firm level to generate a contraction in the economic activity. Khan et al. [2012] and Gomes and Schmid [2010a] instead use equilibrium default to show that credit shocks account for a sizable part of the business cycle fluctuations and generate recessions similar to the recent financial crisis of 2007-2009. Finally, Gomes and Schmid [2010b] develop a model with equilibrium default to explain the relationship between firms' leverage, book assets and stock prices.

Despite the different panorama of questions involved, these papers share the same idea on the role of equilibrium default. In all of them, equilibrium default is just a financial friction, valued in the extent in which is able to produce dynamics which are relevant for investigating phenomena other than default. In other words, it is the instrumental nature rather than the default phenomenon *per se* to be appraised. This paper reverses this logic. In particular it contributes on the literature investigating the phenomenon of corporate default *per se*. In so doing, we restrict our attention on the relationship between the magnitude of default in the economy (default rate) and the price of risk which is associated to it (credit spreads). From a modeling point of view, this paper builds on Arellano et al. [2012], despite the emphasis of the two paper is on completely different questions. In particular, Arellano et al. [2012] focus on the role of financial development on firm dynamics, showing how financial development reduces the differences in leverage and growth rates among large and small firms.

In conclusion, despite this paper is the first documenting the diverging trend between the rise in corporate default rates and constant credit spreads, the increase in default rate is not a new stylized fact. Among others, Campbell et al. [2008] show that corporate bankruptcies have increased by 150% or 300%, depending on how default is measured. A similar upward trend is instead found by Livshits et al. [2012] in the consumer bankruptcies in the United States. They show that, from 1970 to 2002, personal bankruptcies in the United States have increased by around 500%, which is analogous to the number we report on corporate bankruptcies.

3.2 Data

In this section we document the diverging trend between rising corporate default rates and constant credit spreads from the period 1950-2012. This empirical evidence builds upon the contribution of Giesecke et al. [2011] in the measurement of the average default rates and credit spreads of the economy.

3.2.1 Corporate Default Rates

We take the data on US public firms corporate default rates from the Moody's Analytics Default and Recovery Database. The data set covers the credit experiences of over 18,000 corporate issuers that sold long-term public debt at some time between 1920 and 2012[4]. On the one side, an appealing feature of the Moody's data is its broad definition of default which includes not only formal bankruptcy procedures (Chapter 7,Chapter 11) but also informal ones (distressed exchange[5]). We think this is the most relevant definition of default for our analysis, which focuses on economic consequences of default[6]. On the other side, this data set presents some drawbacks. First of all, the index made available to the public by Moody's is issuer-weighted, while a value-weighted index would be a more appropriate measure for gauging the economic impact of default. Second, the sample of firms in consideration (the denominator of the ratio from which we obtain the default rates) is the sample of firms rated by the rating agency, while we would need a broader measure of the public firms population. Third, this index is a global index, which includes not only US firms, contaminating the statistic with foreign default cycles. In conclusion, the index includes also financial and utilities firms, which we would like to depurate, given the different capital structure characteristics. A solution to these problems, would be to use the data of Giesecke et al. [2011], which shares similar properties to the one under study in this paper, and are cleaned of these shortcomings. Hence, until these better data become available, our results should be mostly viewed as suggestive.

Figure 3.2.1 displays the annual issuer-weighted corporate default rates for U.S. public firmsfrom 1950 to 2012. Visual inspection of the picture foregrounds a dramatic upsurge in default rates starting from the 1980's on, following a period of almost no default in the 1950's and 1960's. To test for the presence of a break in the data generating process behind the times series of corporate default rates we apply the Bai and Perron [1998]'s SupLR test statistics. This procedure checks the presence of multiple structural changes,

[4] As of January 1, 2012 approximately 5,000 corporate issuers held a Moody's long-term bond, loan, or corporate family rating, see Moody's [2012], p. 16.

[5] A distressed exchange is one of three events which Moody's defines as a default for the purpose of its default rate statistics. It is any of the following two events: 1) the issuer can make a tender offer, agreeing to pay cash for all or a portion of an outstanding debt security, usually at a price above the trading price, but well below the face amount; 2) the issuer can make an exchange offer, through which an offer is made to substitute the current outstanding securities for a new package of securities which may include: cash, new bonds, stocks, other securities, or a combination thereof, see Moody's [2012].

[6] Financial intermediaries care about the ultimate economic consequence of a delinquent loan, and not about the form of legal bankruptcy or informal default that the debt obligation might have turned into.

occurring at random dates. The test statistic is obtained by running an OLS regression. We test for the existence of at most three break dates. Following Carvalho and Gabaix [2013], we assume that every date T lies in a range $[T_1, T_2]$, with $T_1 = 0.2n$ and $T_2 = 0.8n$, where n denotes the sample size. The choice of a 20% trimming parameter is recommended by Bai and Perron [2006] to reduce the size distortions which is present when allowing for serial correlation in the error.

As a data generator process for the default rates, we consider a first order autoregressive model plus a constant. We run two different test to check for a break in the constant and a joint break in the constant and in the autoregressive coefficient. Table 3.2.1 shows that we reject the null hypothesis of no break for both cases at a 5% significance level. Either case, the SupLR test statistic indicates the existence of a single break, which is estimated at 1983 and 1984. We find statistical evidence of these breaks even after controlling for lagged GDP growth rates and lagged stock returns volatility. These tests tell us that default rates *did* change their dynamics in the early 1980's. Hereafter, we follow the vast literature on the Great Moderation that indicates the existence of a break in the volatility of US GDP growth around 1984[7]. We will then compare two intervals of time, one going from 1950 to 1983 and the second from 1984 to 2012[8]. In Table 2 we report the mean values of the corporate default rates over the two intervals of time. Default rates rose from an average value of 0.3% during the period 1950-1983 up to 1.7% over the last thirty years. This corresponds to a 467% increase in average default rates. Surprisingly, this number almost equals the 500% increase in consumer bankruptcy documented over the same time period by Livshits et al. [2012].

3.2.2 Corporate Credit Spreads

We measure the intensity of corporate default risk using the default rates of the public firms in the economy. Accordingly, we would need an analogous measure of the average price of bond risk. Unfortunately, such a series does not exist. As in Giesecke et al. [2011], we choose the series of spread of a hypothetical average bond, which is considered to be within the Aaa and the Baa credit rating. Therefore, we compute the spreads as the difference between Moody's Baa and Aaa Seasoned corporate all firms bond yields, and available at the FRED database[9]. Our implicit assumption is that the Baa bond proxies for the risky asset in the economy and the Aaa bond is the risk-free asset. We argue that this credit spread is the relevant measure for this analysis. First of all, Baa and Aaa rated corporate bonds belong to the investment-grade class. This class is the most representative form of corporate bond in terms of bond issuance[10] (supply side) and have peculiar liquidity properties[11]. The fact that both the risky asset and safe asset belong to the same class allows us to control (in the data) for common shift in the supply of liquidity for investment-grade bonds (demand side).

[7] Among others, McConnell and Perez-Quiros [2000], Stock and Watson [2002], Carvalho and Gabaix [2013] and reference therein. In particular, Stock and Watson [2002] document a wide-spread decline in aggregate volatility, analysing the time series of 124 macro variables since 1960.

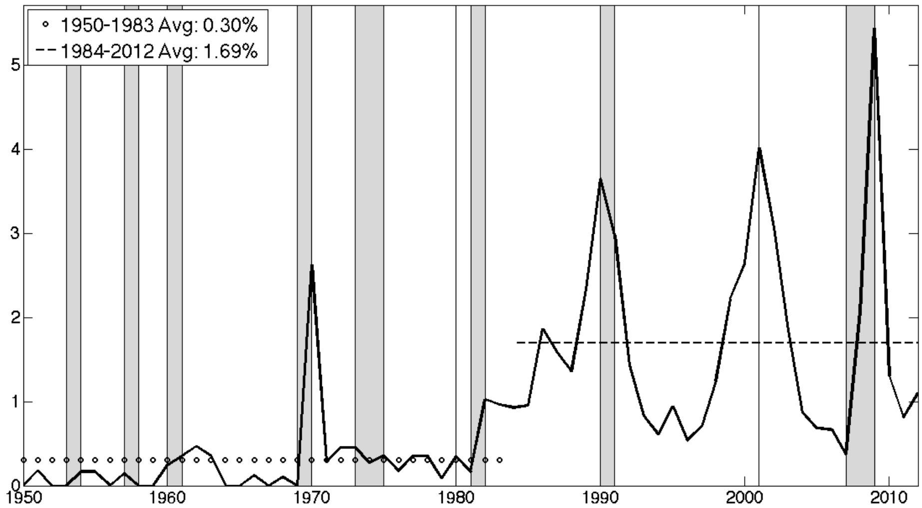
[8] The results of the paper do not change when considering 1983 as the break date.

[9] For more information about the series, see Appendix 3.B.3.

[10] Investment grade bonds account for 2/3 for issuance volume in 1996, and more than 90% in 2006, see Bessembinder and Maxwell [2008], p.28

[11] "For regulated financial service firms, such as banks and life insurance companies, required reserves are greater for noninvestment grade bonds. Further, many financial institutions, including pension and mutual funds, face restrictions on amount of non-investment grade debt they can hold". Bessembinder and Maxwell [2008], p.5.

Figure 3.2.1: Corporate Default Rate



Note: this graph plots the annual corporate default rate (in percentage points) in the United States from 1950 until 2012. The circle line plots the average annual corporate default rate from 1950 until 1983, while the dashed line plots the average annual corporate default rate from 1984 until 2012. Shaded area denotes recession.

Table 3.2.1: Break Test for Default Rates

	$Default_t = a + \rho Default_{t-1} + \epsilon_t$	
	H_0: No Break in a	H_0: No Break in a and ρ
SupLR Stat	8.70	18.14
5% Critical Values	8.22	10.98
Null of No Break	Reject	Reject
Estimated Break Date	1983	1984

Note: this table reports the results of the Bai and Perron [1998] structural break test on annual default rates given by Moody's Default and Recovery Database. We assume that default rates follow an AR(1) process and we test two null hypotheses: either no break just in the constant or no break in both the constant and the autoregressive parameter. The table reports the test statistics (SupLR stat), the 5% Critical Values, whether the test reject or accepts the null hypothesis, and the estimated date of the break in case the null is rejected.

The reason why we preferred the Aaa corporate bond to the Treasury-Bill yield as a proxy of the risk-free rate is manifold. First of all, we study the relation between the dynamics of corporate default and the risk-

Table 3.2.2: Average Default Rates

1950-1983	1984-2012	Δ 1984-2012/1950-1983
0.3%	1.7%	+467%

Note: this table reports the average of annual default rates given by Moody's Default and Recovery Database, over two different periods, from 1950 until 1983, and from 1984 until 2012.

based-differential in the firms cost of financing. Accordingly, an homogeneity argument would support the choice of a *firm* safe corporate bond yield as a proxy of the risk free rate. Moreover, our explanation of the joint dynamics is based on a structural break in the *firms* cost of financing. While we can empirically support that financial development has affected the *firms* cost of debt financing, we cannot claim the same for the government cost of debt financing. Therefore we would unsoundly model the impact on a leg of the credit spread, missing the fact that financial development affects both the cost of risky firm and *safe* firms. Secondly, we can safely affirm that Aaa corporate bonds and Treasury Bills are different securities. Apart from sharing the same rating class, they do not have much in common: they display different market microstructure, taxation, and they are exposed to different sources of risk[12]. All these aspects translate in an average credit spread between Aaa bonds and Treasury Bills amounts to 84 basis points (bp) over the period 1950-2012, which cannot be explained by a simple default risk story we are proposing here[13].

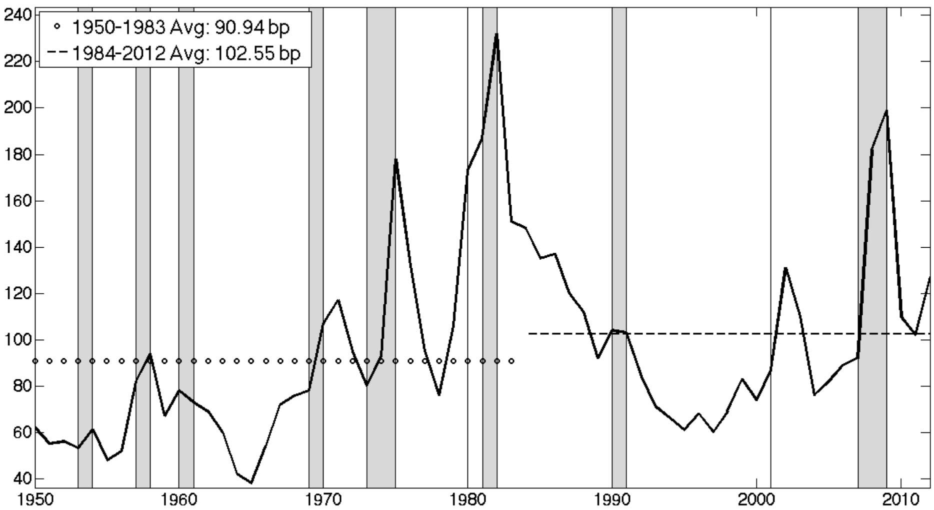
Figure 3.2.2 plots our series of credit spreads, measured in basis points. We can observe how credit spreads were low in the 1950's and 1960's, before peaking up to 232 bp in the 1982. From the 1980's on, credit spreads have been declining to values comparable to the one of 1950's and 1960's. Concomitant with the last financial crisis, credit spreads hike up to 199 bp in the 2009.

As above, we apply the Bai and Perron [1998]'s test to check for structural changes in the credit spreads. Again, to comply with the assumption of the Bai-perron test, we proxy the credit spreads process as a first-

[12]First, Corporate bonds are (an order of measure) less liquid than Treasury bill. The Treasury bill Average Daily Trading Volume in the U.S. Bond Markets in 2001 amount to 297.9$ billions compared to only 17.9$ billions for the whole corporate bond sector. In 2006 the volumes of T-bill almost doubled (524.7$), while the volumes of corporate bonds raises only to 22.7$, see Bessembinder and Maxwell [2008], p. 29. Second, Corporate bond yields are subject to state taxation, while U.S. Treasury securities are exent. Longstaff [2011] shows that tax risk is an important determinant in the pricing of assets. Third, other than the common default risk and liquidity risk, sovereign bonds present a sizeable recovery rates risk. Due to the high uncertainty which characterise enforcement of international debt contracts sovereign bonds display a sizeable heterogeneity in the recovery rates. For example, the credit loss of the 1983-1986 debt restructuring in Argentina was 30%, while the one of the 2001-2002 crisis amounted to 72%. The other major sovereign default crisis, which involved Russia in the August of 1998, was characterized by a credit loss of 63%. On the contrary, recovery rates of Moody's Aa and Baa corporate bonds are stable around 40%, see Moody's [2012], p.26.

[13]Huang and Huang [2012] show that the expected Aaa-Treasury Bill spread should be around 1 bp, given the 0.03% expected 5-year average cumulative credit loss of Aaa corporate bonds.

Figure 3.2.2: Corporate Credit Spreads



Note: this graph plots the annual corporate credit spread (in basis points) in the United States from 1950 until 2012. The circle line plots the average annual corporate credit spread from 1950 until 1983, while the dashed line plots the average annual corporate credit spread from 1984 until 2012. Shaded area denotes recession.

order autoregressive model plus a constant, and - in conclusion - we test whether there is a break either just in the constant or both in the constant and the autoregressive parameter. Table 3 shows that we cannot reject the null of no break in either cases. Controlling for lagged GDP growth rates[14] and lagged stock returns volatility, testing for breaks using quarterly data and using the Chow test with 1983 or 1984 as pre-determined break date does not alter our finding[15]. Table 4 reports the mean values of the credit spreads over the two periods of interest, 1950-1983 and 1984-2012. From the 1950's to the 1970's, the average value of the credit spread was 91 bp. This average barely changed over the following three decades, reaching 102 bp. On the ground of the evidence provided by the break tests, we interpret this 11 bp increase as not statistically significant. To check whether these 11 basis points are economically significant, we follow Giesecke et al. [2011] by using a back-to-the-envelope to estimate the average annual credit losses, assuming 50% recovery rate[16]. We find that a 1.4% increase in default rates should have pushed up credit spreads by 70 bp instead of the 11 bp observed in the data. Secondly, despite default rates have increased to a record number of 5.45% in the recent financial crises, the credit spread reached a peak of 199 bp. Yet, the global maximum over the over-all period in consideration, equals 232 bp and was reached in 1982 with a default rate of *only* 1.16%.

[14]Gomes and Schmid [2010a] investigates the endogenous link between macro aggregates and credit spreads.

[15]We apply the Chow test to quarterly data using as a pre-determined data any quarter between 1982:1 and 1984:1, and in all cases we reject the presence of a break at the 5% significance level.

[16]The back-to-the-envelope estimate multiplies the physical probability of default to the expected loss upon default. We consider a 50% recovery rate, which is the average senior unsecured recovery rates on investment grade bond for the period 1982-2011, see Moody's [2012], p. 9. As a result, a 1% default rate translates into 50 basis points.

Table 3.2.3: Break Test for Credit Spreads

	$Spread_t = a + \rho Spread_{t-1} + \epsilon_t$	
	H_0: No Break in a	H_0: No Break in a and ρ
SupLR Stat	5.68	5.62
5% Critical Values	8.22	10.98
Null of No Break	Accept	Accept
Estimated Break Date	-	-

Note: this table reports the results of the Bai and Perron [1998] structural break test on annual credit spreads, measured as the difference (in basis points) between Moody's Baa and Aaa bond yields. We assume that credit spreads follow an AR(1) process and we test two null hypotheses: either no break just in the constant or no break in both the constant and the autoregressive parameter. The table reports the test statistics (SupLR stat), the 5% Critical Values, whether the test reject or accepts the null hypothesis, and the estimated date of the break in case the null is rejected.

This over period max-max comparison provides further economical support of our claim that something has structurally changed in the dynamics of default rates and credit spreads. As a conclusion, the change in average credit spreads over the two periods is insignificant from both an economic and a statistical point of view.

3.2.3 Diverging Trend

In summary, starting from the early 1980's default rates rose by 467% while credit spreads kept constant. We refer to this evidence as the diverging trend between default rates and credit spreads. Longstaff et al. [2005] find that default risk explains 71% of the Baa bond yields. Therefore, a 467% increase in default rates should come at a neat rise in the credit spreads. In addition, even if actual average default rates in both periods are low in absolute value, such a steep increase in default rates should be mirrored in credit spreads for two reasons. First, Almeida and Philippon [2007] show that the risk-adjusted cost of default is four-five times larger than what the physical bankruptcy rates would suggest. Indeed, default is more likely to occur in bad times, which makes risk-averse agents to care more about financial distress than is suggested by physical credit losses. Second, though the average default in the last thirty years equals 1.7%, now financial distress has become more likely for the median firm too. In this sense, the rise in default rates cannot be diversified away and should, therefore, be translated in the pricing of debt.

Table 3.2.4: Average Credit Spreads

1950-1983	1984-2012	Δ 1984-2012/1950-1983
91 bp	102 bp	+11 bp

Note: this table reports the average of annual credit spreads, measured as the difference (in basis points) between Moody's Baa and Aaa bond yields, over two different periods, from 1950 until 1983, and from 1984 until 2012.

3.2.4 Financial Development

In this paper we quantitatively investigate how financial development can affect average default rates and credit spreads, by influencing the economic decision of all the firms. Following the seminal papers of King and Levine [1993] and Rajan and Zingales [1998], a vast literature attempted to study the interaction between financial development and the real economy, an idea that actually traces back to Schumpeter (1911).

We focus on the process of deregulation and innovation that characterized the financial sector during the 1970s. This decade saw the introduction, among others, of ATMs, phone transfers for savings balances at commercial banks, the International Banking Act, the modification on the Regulation Q on the banking system, the Financial Institutions Regulatory and Interest Control Act, the Electronic Fund Transfer Act, the 1979 Bankruptcy Reform Act, NOW (negotiable order of withdraw) accounts, the securitization of debt collateralization, the introduction of the Securities Protection Act and the introduction of Asset Backed Securities (ABS), which has recently became the first source of funding for U.S. corporate firms, undertaking corporate bonds.

The deregulation in the financial sector has improved the access to credit for corporate firms, especially the small ones, and decreased the cost of external financing. Nowadays firms can borrow more and cheaper than 30 years ago. This view is supported by the empirical evidence provided by Jayaratne and Strahan [1996] and Demyanyk et al. [2007], among others. Accordingly, in our analysis we will model financial development in a reduced form, as an exogenous reduction in the fixed costs of borrowing.

3.3 The Model

3.3.1 Environment

In the economy there are two types of agents: firms and credit intermediaries. Firms have decreasing returns to scale production technologies and experience in each period a persistent idiosyncratic productivity shock and an i.i.d stochastic fixed cost of operation. They are run by risk neutral managers which maximize the expected discounted stream of dividends. Firms articulate in two types: incumbents and entrants. At each point of time, there is a distribution of heterogeneous incumbents, which are defined as the producing firms of the economy. Incumbents finance investment and dividends using internal and external funds:

retained profits, new equity issuance and one-period non-contingent loans from the credit intermediaries. Incumbents can renege on their obligations and default. The presence of default risk generates endogenous borrowing constraints for the firms and makes loans' interest rates to be firm-specific. Less efficient firms and/or firms with less collateral face tighter borrowing constraints and access to loans at higher interest rates than more efficient firms and/or firms with more collateral.
Every period a mass of firms enters the economy and starts the production with a time-to-build lag. Entrants solve a problem identical to the incumbents with the difference that they resort uniquely to external funds. There is also a competitive financial sector. Each financial intermediary offers a menu of loan sizes and interest rates to firms wherein each loan makes zero expected profits. When a firm defaults, creditors can seize its assets and profits net of a liquidation loss.

3.3.2 Firms

In the economy there are two types of firms: incumbents and entrants. Henceforth we denote with the i subscript an incumbent firm, while e stands for an entrant firm. We omit the subscript when the distinction is not necessary.
Firms use capital $k \in \mathbf{K} \subset \mathbb{R}_+$ to produce an homogeneous consumption good $y \in \mathbf{Y} \subset \mathbb{R}_+$ using a decreasing returns to scale technology[17,18]

$$y = xk^{\alpha} \tag{3.3.1}$$

where $\alpha \in (0,1)$ captures the degree of concavity of the production function and x is an uninsurable idiosyncratic shock. The idiosyncratic productivity $x \in \mathbf{X} \subset \mathbb{R}_+$ follows a first-order Markov processes whose transition function is $p_x(x'|x)$. In each period firms incur in a stochastic fixed cost of operation. The operating profits before interest and depreciation are defined as:

$$\pi = xk^{\alpha} - \chi \tag{3.3.2}$$

where $\chi \in \chi \subset \mathbb{R}_+$ is the i.i.d. fixed cost of operation drawn from the cumulative distribution $H(\chi)$. This shock is intended to create a link between negative cash flows and the firms' decision of going bankrupt. Without this feature, firms would always have non-negative profits. However, in the data defaulting firms experience negative profits. Physical capital depreciates at a rate $\delta \in (0,1)$ and accumulates with the law of motion

$$k' = (1-\delta)k + i \tag{3.3.3}$$

where k and k' denotes, respectively, the actual and next period stock of physical capital and i is the capital investment.
Entrants finance dividends and investment with one-period non-contingent loans and new equity issuance. Incumbents can resort, in addition, to retained profits. Because of limited enforceability, firms can renege

[17] Diminishing returns to scale at the firm-level may be explained with the span of control models of Rosen [1982] and Lucas [1978].

[18] Decreasing returns to scale technologies and perfect competition prevent the most productive firms from taking over the market completely and allow for the existence of heterogeneity in equilibrium. Since firms can be replicated, returns to scale are constant at the aggregate level.

on their debt. Then, loan contracts depend on those firms' characteristics that are informative about the default probability and the loss given default. When an incumbent firm defaults, it partially meets its obligations with the creditors. In such a case, the firm is liquidated and the creditors seize both its profits and undepreciated capital

$$L(k, x) = \max\{(1 - \psi)\left(\pi + (1 - \delta)k\right), 0\} \tag{3.3.4}$$

suffering a liquidation clearance loss $\psi \in (0, 1)$. The recovery rate is then $L(k, x)/b$, where b refers to the firm outstanding debt.

Every period, after observing the realization of the shocks, incumbents choose whether to enter into a one-period non-contingent loan contract. A contract formalizes in a 4-tuple $\left(x_i, k'_i, l'_i, r'_i\right)$, which delivers a loan l'_i whose repayment value is $b'_i = \left(1 + r'_i\right) l'_i$, to firms with idiosyncratic efficiency x_i and future stock of physical capital k'_i. Contracts $\left(x_i, k'_i, l'_i, r'_i\right)$ belong to a set of debt schedules $\Omega\left(x_i, k'_i, l'_i\right)$. This specification highlights the dependence of interest rates on three firms' *key characteristics*: 1) the productivity, 2) the size of assets and 3) the size of the loan. If the actual productivity is high, next period productivity is more likely to be high[19]. This decreases the probability of default and the interest rates. Similarly, firms with more capital have a larger collateral and therefore lower interest rates. Finally, larger loans increase the probability of default, implying a higher interest rate. It is worth noticing that the future levels of capital and outstanding debt are chosen at the same time, and they jointly determine the interest rate required by the credit intermediaries. Entrants face the same loan contracts with the difference that their debt schedules and interest rates do not depend on the idiosyncratic shock x_e.

Firms issue new equity when their dividends d are negative. The equity issuance comes at an additional proportional cost, $\gamma > 1$. The total cost of distributing dividends $d \in \mathbb{R}$ is then

$$g(d) = d\mathbb{I}_{\{d \geq 0\}} + (\gamma d)\mathbb{I}_{\{d<0\}}$$

where $\mathbb{I}_{\{y\}}$ is an indicator function that takes value 1 when y is true. The implication of the issuance cost is twofold. It prevents firms from distributing dividends and raising equity at the same time and it does not allow firms to issue as much equity as they need to circumvent the financial frictions due to bonds' limited enforceability[20]. Then, the equity issuance cost makes firms to prefer the use of retained profits and debt to equity, in accordance with the pecking order theory.

Firms can also save in the market portfolio of corporate bonds ($l' < 0$). Since the idiosyncratic uncertainty washes out in the aggregate, the gross return on the market portfolio of corporate bonds is the risk-free rate $1 + r_F$. Thus, the repayment value $b \in \mathbf{B} \subset \mathbb{R}$, is

$$b = \left([1 + r]\mathbb{I}_{\{l>0\}} + [1 + r_F]\mathbb{I}_{\{l<0\}}\right) l$$

[19] This argument holds as long as firms' idiosyncratic productivity shocks are persistent. In case of i.i.d productivity shocks the interest rate would not depend anymore on the *actual* x, as it happens for the fixed cost shock χ.

[20] The presence of the equity issuance cost and the bankruptcy deadweight loss make the Modigliani and Miller [1958] theorem not to hold in this framework.

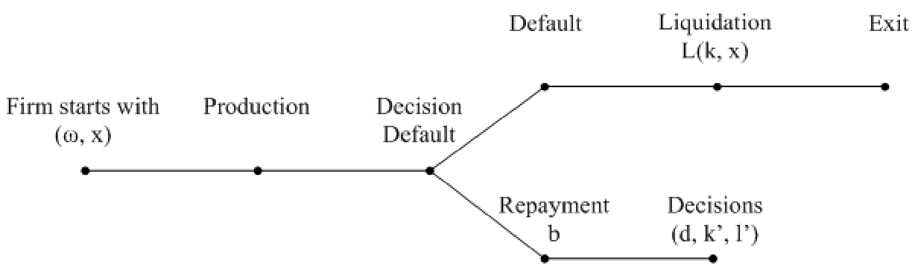
Incumbents

An incumbent begins with an amount of net-wealth $\omega_i \in \mathbf{W} \equiv \mathbf{X} \times \mathbf{K} \times \chi \times \mathbf{B} \subset \mathbb{R}^3_+ \times \mathbb{R}$, which is a by-product of its holdings of physical capital and outstanding debt, that is

$$\omega_i = \pi_i + (1-\delta)k_i - \Big([1+r_i]\,\mathbb{I}_{\{l_i>0\}} + [1+r_F]\,\mathbb{I}_{\{l_i<0\}}\Big) l_i$$

At each point of time, there is a large measure λ of incumbents, which are defined as the set of firms that either were operating or entered in the previous period. λ is the probability measure over (ω_i, x_i), defined on the Borel algebra $\mathcal{J}$ generated by the open subset of the product space $\mathbf{J} = \mathbf{W} \times \mathbf{X} \subset \mathbb{R}^4_+ \times \mathbb{R}$.
We assume that an incumbent first observes the realization of the idiosyncratic productivity shock x_i and the stochastic fixed cost of operation χ_i, and then produces. At this point, each firm maximizes the expected present value of future profits in a two stage decision problem. First, a firm decides whether to default or not. The default implies the exit of the firm and an outside opportunity of not operating equals zero. Therefore, firms default whenever their continuation value is negative. Second, if the firm does not default, it finances the entire value of its outstanding liabilities $(1+r_i)l_i$, and decides the amount of dividends to distribute d_i, the new level of physical capital k'_i, and the new level of debt l'_i, given the debt schedules $\left(x_i, k'_i, l'_i, r'_i\right) \in \Omega\left(k'_i, l'_i, x_i\right)$. Figure 3.3.3 summarizes the timing of the model.

Figure 3.3.3: Timing of the Model



The states of the economy for an incumbent firm are, therefore, (ω_i, x_i). The incumbents' problem can be written as

$$V_i(\omega_i, x_i) = \max_{\phi_{D,i}\in\{0,1\}} (1-\phi_{D,i})V^c_i(\omega_i, x_i) \tag{3.3.5}$$

where $\phi_{D,i} = \phi_D(\omega_i, x_i)$ is an indicator function that takes value $\phi_{D,i} = 1$ in case of default, and $V^c_i(\omega_i, x_i)$ denotes the continuation value of an incumbent firm which does not default,

$$V^c_i(\omega_i, x_i) = \max_{d_i, k'_i, l'_i} d_i + \beta\mathbb{E}_{H(\chi'_i), x'_i|x_i}\left[V_i(\omega'_i, x'_i)\right] \tag{3.3.6}$$

$$\text{s.t. } g(d_i) = \omega_i + l'_i - k'_i$$

$$\omega'_i \equiv \omega'_i(k'_i, l'_i, x'_i, \chi'_i) = \pi'_i + (1-\delta)k'_i - \Big(\left[1+r'_i\right]\mathbb{I}_{\{l'_i>0\}} + [1+r_F]\,\mathbb{I}_{\{l'_i<0\}}\Big) l'_i \tag{3.3.7}$$

$$\left(x_i, k'_i, l'_i, r'_i\right) \in \Omega(k'_i, l'_i, x_i) \tag{3.3.8}$$

where: 1) β denotes the subjective time discounting rate of the firm's manager; 2) $\mathbb{E}_{H(\chi_i'),x_i'|x_i}$ denotes the expected value over the independent processes of χ_i' and x_i', where the realization of x_i' is conditional on x_i; 3) equation (3.3.7) denotes the law of motion of firm's net worth. ω_i' is a random process which inherits the first-order Markov property from the idiosyncratic productivity shock x_i', augmented by the independent i.i.d process of χ_i'. Formally ω_i' follows the transition function $p(x_i'|x_i)H(\mathrm{d}\chi_i')$[21].

Analogously to the analysis of entry and exit in Hopenhayn [1992], we can describe the optimal default policy as a threshold on the idiosyncratic productivity shock. Here the definition of the threshold is complicated by the dependence of the continuation value of the incumbents on both[22] the idiosyncratic productivity shock and the wealth. Using the weakly increasing property of the continuation value on both arguments, Khan et al. [2012] prove that for each level of ω_i there exists a schedule $\underline{x}_i = x(\omega_i)$ such that a firm with net-wealth ω_i defaults if and only if its productivity is lower than $\underline{x}_i$. Such a threshold $\underline{x}_i$ is defined as the value of x wherein $V^c(\omega_i, \underline{x}_i) = 0$.

Likewise, it might be shown that when firms default, their net-wealth is negative. Intuitively, when the net-wealth is non-negative, a firm is always able to pay back the debt without resorting to any additional external fund. In turn, this implies that the liquidation value (3.3.4) which the creditors seize out of defaulted firms is always less than the due repayment values of the loans. This result guarantees that creditors always incur in a loss when firms default.

Before concluding, it is worth noticing that the negative net-wealth is a necessary but not sufficient condition for the firm to default. Indeed, a firm with negative net-wealth can find optimal not to exit and decide to issue equity and roll over debt to fund its operations.

Entrants

The model features exogenous entry. At each point of time there is a mass Ξ_t of firms which enters in the economy, merely substituting the measure of firms which default. Production takes place with a lag, as a time-to-build restriction. The entrants begin the period with an amount of physical capital k_e. They then decide the amount of dividends to distribute d_e, the new level of physical capital k_e', and debt l_e' (or savings if $l_e' < 0$) with $(k_e', l_e', r_e') \in \Omega\left(k_e', l_e'\right)$ to maximize the expected present value of future profits. The entrants can also decide to raise equity at the proportional cost γ. Once entrants have solved for their optimal choices, they draw next-period idiosyncratic shock x_e' from a cumulative distribution $G\left(x_e'\right)$. The state of the economy for an entrant is then k_e. Hence, the entrants' problem can be written as

$$V_e(k_e) = \max_{d_e,k_e',l_e'} d_e + \beta\mathbb{E}_{H(\chi_e'),G(x_e')}\left[V_i(\omega_e', x_e')\right] \tag{3.3.9}$$

$$\text{s.t. } g(d_e) = k_e + l_e' - k_e'$$

$$\omega_e' \equiv \omega_i'(k_e', l_e', x_e', \chi_e') = \pi_e' + (1-\delta)k_e' - \left(\left[1 + r_e'\right]\mathbb{I}_{\{l_e'>0\}} + \left[1 + r_F\right]\mathbb{I}_{\{l_e'<0\}}\right)l_e' \tag{3.3.10}$$

$$\left(k_e', l_e', r_e'\right) \in \Omega(k_e', l_e') \tag{3.3.11}$$

[21] Where we assume that the pdf of $H(\chi_i')$ exists and it is atomless.

[22] In Hopenhayn [1992], the continuation value depends only on the productivity shock.

3.3.3 Credit Intermediaries

In the economy there is a competitive financial sector which lends to firms (or borrows from firms, in case they save). The credit intermediaries offer a menu of loan sizes and interest rates, wherein each loan makes zero profits. For each loan the intermediaries have to pay a *fixed cost* ζ. As suggested by Arellano et al. [2012], the fixed credit cost can be interpreted as any financial intermediation cost that creditors incur when issuing a loan, as costs to obtain information about firms' default probability and overhead costs. The higher the value of ζ, the larger the costs firms incur in borrowing from the credit intermediaries, the less developed is the financial sector of the economy. Following Arellano et al. [2012], we consider this cost as a proxy for financial development.
The credit intermediaries price firms bonds by defining debt schedules which contingent on firm characteristics. The latter captures the probability of default and the amount of insurance in case of default. Formally, the credit intermediary offers a set of incumbent-specific contracts $\left(x_i, k_i', l_i', r_i'\right) \in \Omega\left(k_i', l_i', x_i\right)$ which read: in absence of arbitrage opportunities, the incumbent-specific (x_i, k_i') interest rate r_i' associated to a required amount of loan l_i' is defined by the zero profit (break-even) condition of the intermediaries $\Omega\left(k_i', l_i', x_i\right)$:

$$\left(l_i' + \zeta\right)(1 + r_F) = \mathbb{E}_{H(\chi_i'), x_i'|x_i}\left[(1 - \phi_{D,i})(1 + r_i')l_i' + \phi_{D,i}L(k_i', x_i')\right] \tag{3.3.12}$$

where ζ denotes the fixed cost of borrowing and $L(k_i', x_i')$ is the liquidation value of the firm in case of default, as defined in (3.3.4). In case of an entrant, the mapping is identical but for the expectation, which is taken unconditionally over the idiosyncratic shock x_e.
The availability and the interest rates of each loan depend on the default risk, on the amount of insurance provided by the expected liquidation value and on the borrowing costs ζ. While the first two channels generate endogenous borrowing constraints which are firm specific, the presence of fixed credit costs limits all firms access to credit. As pointed out above, the fixed cost has a further asymmetric effect: small and less efficient firms suffer disproportionately more from it.

3.4 Characterization of the Equilibrium

3.4.1 Definition of Equilibrium

A recursive equilibrium in this economy is given by the optimal choices of the incumbents $(\phi_{D,i}, k_i', l_i', d_i)$, optimal choices for the entrants (k_e', l_e', d_e), an exogenous risk-free rate r_F and the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, $\left(k_e', l_e', r_e'\right)$, such that:

1. given the exogenous risk-free rate r_F, the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, $\left(k_e', l_e', r_e'\right)$ satisfy the zero ex-ante profit condition of the credit intermediary (3.3.12)[23], for any choice of (k_i', l_i') and (k_e', l_e');
2. given the exogenous risk-free rate r_F and the firm-specific contracts $\left(x_i, k_i', l_i', r_i'\right)$, the incumbent firms choose $(\phi_{D,i}, k_i', l_i', d_i)$ to maximize their problem described in (3.3.5);

[23] Recall the variant of the zero ex-ante profit condition of the credit intermediary (3.3.12) for the entrant requires to use in the expectation the unconditional distribution $G(x_e')$ to determine the next period idiosyncratic shock.

3. given the exogenous risk-free rate r_F and the firm-specific contracts $\left(k'_e, l'_e, r'_e\right)$, the entrant firms choose (k'_e, l'_e, d_e) to maximize their problem described in (3.3.9);

4. given the exogenous risk-free rate r_F, the firm-specific contracts $\left(x_i, k'_i, l'_i, r'_i\right)$, $\left(k'_e, l'_e, r'_e\right)$, the optimal choices of the incumbents $(\phi_{D,i}, k'_i, l'_i, d_i)$ and the optimal choices for the entrants (k'_e, l'_e, d_e), the law of motion of the distribution of firms is given by

$$\lambda(\omega'_i, x'_i) = \int \left(1 - \phi_{D,i}\right) Q\left(\left(\omega'_i, x'_i\right), (\omega_i, x_i)\right) H(\chi')p_x\left(x'_i|x_i\right) \lambda(\mathrm{d}\omega_i, \mathrm{d}x_i)$$
$$+ \int \phi_{D,i} Q_e\left(\omega'_e, x'_e\right) H(\chi'_e)G\left(x'_e\right) \lambda(\mathrm{d}\omega_i, \mathrm{d}x_i)$$

where $Q\left(\left(\omega'_i, x'_i\right), (\omega_i, x_i)\right)$ denotes a transition functions such that

$$Q\left(\left(\omega'_i, x'_i\right), (\omega_i, x_i)\right) = \begin{cases} 1, & \text{if } \omega'_i(\omega_i, x_i) = \omega'_i,\ \ x'_i(x_i) = x'_i \\ 0, & \text{if otherwise} \end{cases}$$

The same applies to $Q_e(\omega'_e, x'_e)$.

3.4.2 The Role of Financial Development

In this section we investigate analytically the effect of changes in the borrowing cost ζ. The idea is to isolate in the simplest framework the effects of financial development on the borrowing constraints, and the leverage in the economy. We consider a simple economy without uncertainty, where there is a continuum of firms which are born with different idiosyncratic productivity $\bar{x}_i$, henceforth constant. Incumbents do not suffer stochastic fixed cost of operation. Entrants are endowed with no capital and debt. Output is produced using capital, which fully depreciate each period. Firms cannot save or issue equity. The following Propositions follow the results in Arellano et al. [2012]. The interested reader can refer to Appendix 3.A for the detailed proofs.

Proposition 3.4.1. *In this economy there is a unique equilibrium which is characterized as follows. In equilibrium:*

1. *The policy functions of the firms are constant,* $(\phi^*_{D,i}, k'^*_i, l'^*_i, d^*_i)$
2. *Firms do not default.* $\phi^*_{D,i} = 0$
3. *Firms can borrow at the risk free rate, corrected for the fixed cost of borrowing. In formula:*

$$(1 + r^*_i)l^*_i = (1 + r^*)l^*_i + (1 + r^*_i)\zeta \tag{3.4.13}$$

4. *Firms demand capital up to equalize their marginal product to the risk-free interest rate. Let me name this level of capital as the first-best level of capital,* $k_{fb,i}$*:*

$$k_{fb,i} = \left(\frac{\alpha \bar{x}_i}{1 + r_i}\right)^{\frac{1}{1-\alpha}} \tag{3.4.14}$$

Notice there is a one-onto-one increasing relationship between $k_{fb,i}$ *and* $\bar{x}_i$*.*

5. *Firms are subject to endogenous borrowing constraints.*

The endogenous borrowing constraints arise from the necessity of making *incentive compatible* for the firms not to default. In particular:

Proposition 3.4.2. *A no defaulting equilibrium strategy for a firm i is sustained for level of debt $l_i^* \in [0, l_{D,i}]$, where $l_{D,i}$ represent the equilibrium firm-specific debt limit and it is defined as:*

$$l_{D,i} = \frac{1 + r_i - \alpha}{r_i \alpha} k_{fb,i} - \frac{(1 + r_i)}{r_i} \zeta \tag{3.4.15}$$

We name this level of debt, firm specific debt-limit, and it is defined as the level of debt for which it is incentive compatible for the firm not to default.

The endogenous nature of the debt-limit rationalizes the label endogenous borrowing constraint. Proposition 3.4.3 investigates the sensitivity of $l_{D,i}$ to financial development and (through the optimal choice of capital), to the level of idiosyncratic productivity of the firms.

Proposition 3.4.3. *In equilibrium:*

- $\frac{\partial l_{D,i}}{\partial \zeta} = -\frac{1 + r_i}{r_i} < 0$*: the debt-limit is increasing in the level of financial development. The lower the level of financial development (the higher is ζ), the lower is the level of debt for which the firm is indifferent whether to default or not.*
- $\frac{\partial l_{D,i}}{\partial k_{fb,i}(\bar{x})} = \frac{1 + r_i - \alpha}{r_i \alpha} > 0$*: the debt-limit is increasing in the optimal choice of capital, which depends uniquely on the original idiosyncratic efficiency. Then, the higher is the idiosyncratic productivity, the higher is the debt limit.*

In equilibrium the leverage evaluated at the debt-limit can be expressed as:

$$lev_i = \frac{l_{D,i}}{k_{fb,i}} = \frac{1 + r_i - \alpha}{r_i \alpha} - \frac{(1 + r_i)}{r_i} \frac{\zeta}{k_{fb,i}} \tag{3.4.16}$$

Similarly to what we have just done, Proposition 3.4.4 explores the sensitivity of the leverage, lev_i, to financial development.

Proposition 3.4.4. *In equilibrium:*

- $\frac{\partial lev_i}{\partial \zeta} = -\frac{1 + r_i}{r_i} \frac{1}{k_{fb,i}} < 0$*: the leverage is strictly increasing in the level of financial development. The more developed is the credit intermediation (the lower is the ζ), the higher is the equilibrium leverage of the firms.*
- $\frac{\partial lev_i}{\partial k_i^{fb}(\bar{x})} = \frac{1 + r_i}{r_i} \frac{\zeta}{(k_{fb,i})^2} > 0$*: the leverage is strictly increasing in the amount of capital. The higher is the productivity of a firm, the higher is the optimal level of capital, the higher is the equilibrium leverage.*

3.5 Quantitative Analysis

In this section we study the quantitative implications of financial development on the joint behavior of default rates and credit spreads and on other relevant dynamics of the US economy. To that end, we compute two equilibria whose parameters differ only for the value of the fixed borrowing costs. The first equilibrium is calibrated to proxy the behaviour of some relevant facts of the US economy over the period 1950-1983. The second equilibrium approximates the US economy over the period 1984-2012; it takes as given the estimated and calibrated parameters from the first period *but* for the fixed costs of borrowing, which are lowered. In line with the calibration strategy adopted by Buera and Shin [2013], we discipline this cut by matching the higher leverage of the firms over the period 1984-2012[24]. As mentioned above, the decline in the fixed cost of borrowing is a reduced form way of modelling financial development[25]. This modelling strategy allows us to isolate and study the implications of financial development, and to test whether it might have been a relevant structural explanation of the diverging trend observed in the data.

3.5.1 Calibration

We calibrate the model over the period 1950-1983. In the model, one period corresponds to one year.

Uncertainty in the Economy

In order to proceed we need to impose more structure on the stochastic properties of the uncertainty in the economy: the idiosyncratic productivity shock and the stochastic fixed cost of operation. The idiosyncratic productivity shock of the incumbents follows an AR(1) process, such that

$$x_t = \rho_x x_{t-1} + e_t, \quad e_t \sim N(0, \sigma_e^2) \tag{3.5.17}$$

In the context of the calibration, we transform (3.5.17) into a discrete-state Markov chain, with 2 points in the support, using the standard Tauchen [1986] algorithm. Then, we assume the distribution $G(x)$ from which the entrants draw their first realization of the idiosyncratic shock is a Pareto distribution with exponent c. The choice of such a distribution is in accordance with the empirical evidence that the firms' size distribution is very heavy tailed, see Gabaix [2011] among others. Finally, we assume that the stochastic costs of operation follows an i.i.d. process, where $H(\chi)$ is modeled as a Bernoulli random variable which takes the value χ with probability p_χ and the value 0 with probability $1 - p_\chi$.

Estimated Parameters

Table 3.5.5 reports the estimated parameters and the source whence they are taken. Following Gomes and Schmid [2010b] and Arellano et al. [2012], we set the parameters governing the decreasing returns to scale of the firms' production function to $\alpha = 0.65$[26]. The depreciation rate of capital is set to 10% per year. The risk-free interest rate is set to $r_F = 0.04$ according to the actual value of the annual real interest rate in the United States from 1950 to 1983. As in Arellano et al. [2012], we set the subjective discount rate parameter to $\beta = 0.9605$ per year. This value is slightly lower than its frictionless equilibrium value of $\dfrac{1}{1 + r_F}$, as a proxy of

[24]Buera and Shin [2013] studies the role of financial frictions on the so-called miracle economies. In their calibration, the authors pin down the exogenous size of financial development by matching the evolution of external finance to GDP ratios in the data.

[25]Other examples of models where financial development is modeled as an exogenous reduction of the economy's financial frictions are Buera et al. [2011] and Arellano et al. [2012].

[26]This value is on the lower bound of reasonable parameters for the Cobb-Douglas production function

Table 3.5.5: Estimated Parameters

	A. Firms	
$\alpha = 0.65$	Production Function Returns to Scale	Arellano et al. [2012]
$\delta = 0.10$	Capital Depreciation	Arellano et al. [2012]
$\psi = 0.30$	Bankruptcy Deadweight Loss	Gomes and Schmid [2010a]
$r_F = 0.04$	Real Risk-Free Interest Rate	Data (see Appendix B.4)
$\beta = 0.96$	Time Discounting Parameter	Arellano et al. [2012]
$\rho_x = 0.80$	Idiosyncratic Shock Persistence	Foster et al. [2008]
$\gamma = 0.35$	Equity Issuance Cost	Cooley and Quadrini [2001]
	B. Incumbents	
$p_\chi = 0.06$	Probability Operational Cost	Armenter and Hnatkovska [2011]
	C. Entrants	
$c = 2$	Pareto Exponent	Axtell [2001]

Note: this table reports the values, the description and the source of the estimated parameters.

the impatience of the risk neutral manager. This feature supplements the absence of nontax deducibility of interest rate payments in providing the firms with an incentive to borrow. As in Gomes and Schmid [2010a] we set the bankruptcy deadweight loss to $\psi = 30\%$[27]. The value of the equity issuance costs $\gamma = 0.35$ is borrowed from Cooley and Quadrini [2001]. We take the value of the autoregressive parameter $\rho_x = 0.80$, from Foster et al. [2008], who estimate the production function and the Solow residual at the firm level. In line with the evidence of Axtell [2001] and Gabaix [2011] that the distribution of firm size is heavy tailed, we choose a Pareto exponent of 2. Finally, we set the probability of receiving a positive operational cost to 6% to match the transition rate from positive to negative cash flows for US public firms estimated by Armenter and Hnatkovska [2011].

Calibrated Parameters

There are four parameters to be calibrated: 1) σ_e, the standard deviation of the idiosyncratic productivity shock; 2) χ, the magnitude of the stochastic costs of operation; 3) ζ, the borrowing fixed cost; and 4) k_e, the

[27] Warner [1977] estimates that the direct and indirect costs associated to corporate bankruptcy equal 30% of the book value of the firm.

physical capital endowment of an entrant. Accordingly, we need (at least) four targets. Consistently with the structural break observed in the data, we calibrate our model economy to statistics computed over the period 1950-1983. In principle, these targets should capture relevant information about the process driving the default phenomenon in the US economy. In this spirit, we choose: 1) the average debt to asset ratio, b_i/a_i, where the assets in the model are given by $a_i = x_i k_i^\alpha + (1-\delta)k_i$; 2) the cross-sectional standard deviation of the average ratio of debt over asset; 3) the average default rate from 1950 to 1983; and 4) the growth rate of entrants. Apart from the last target - which is taken from Arellano et al. [2012] - the statistics are computed using Compustat data. Appendix 3.B describes in the detail the construction of the data.

Results of the calibration. Table 3.5.6 reports the value of the calibrated parameters while Table 3.5.7 compares the targets in the data with the one computed in the model. The stochastic operational cost is calibrated to $\chi = 9$. In relative terms, it represents 9% of the assets of the median firm. As far as the targets are concerned, on one hand, the average debt to asset ratio in the model is slightly overestimated at 0.29, compared to the actual value in the data of 0.24. In the model, firms borrow a little too much. On the other hand, the cross-sectional standard deviation of the ratio is much closer to the actual one, with a value of 0.17 compared with the 0.16 in data. The average default rate is perfectly matched: in the model the 0.3% of the firms defaults, as it is in the data. This is a successful matching since it is well known that models with equilibrium default struggle in providing quantitatively reasonable amount of default in the economy. As instance, in Arellano et al. [2012], the average default rate implied by the model is zero. Conversely, this model does not suffer the same weakness. The reason of this relevant difference rests on the introduction of the stochastic fixed cost of operation. Finally, we perfectly match the growth rate of the entrants.

Table 3.5.6: Calibrated Parameters

Parameter	Description
$\sigma_e = 0.65$	Standard Deviation Idiosyncratic Productivity Shock
$\chi = 9$	Stochastic Operational Cost
$\zeta = 0.60$	Borrowing Fixed Costs
$k_e = 145$	Capital Entrants

Note: this table reports the values and the description of the calibrated parameters.

3.5.2 Results: Financial Development and the Diverging Trend

We study the quantitative implications of financial development on the dynamics of default rates and credit spreads. To that end, we exogenously cut from 0.6 to 0.4 the fixed borrowing costs calibrated in the first equilibrium. We discipline this reduction by matching the higher ratio of total debt to asset observed over the

Table 3.5.7: Calibration Targets

Target	Data	Model
Average Debt to Asset Ratio	0.24	0.29
Standard Deviation of Debt to Asset Ratio	0.16	0.17
Average Default Rate	0.3%	0.3%
Growth Rate Entrants	0.95%	0.95%

Note: this table reports the targets of the calibration exercise. The Debt to Asset Ratio is computed in the model as the total amount of debt over the sum of firm's profits and undepreciated capital, and in the data as the book value of assets over the sum of long-term debt and debt in current liabilities.

period 1984-2012. This structural change leads a median firm leverage ratio of 0.36 in the second equilibrium, close to the 0.33 measured in the data. To confirm the plausibility of the magnitude of these values, note that the ratio of the fixed borrowing costs over the loan value for the median firm is 0.5% in the first equilibrium and 0.13% in the second equilibrium. Those figures are in line with the results in Altinkilic and Hansen [2000], which study a panel of 628 industrial firms from 1990 until 1997 and find that the fixed cost of debt issuance for public debt equals on average around 0.1% of the debt principal. As a byproduct, this result provides a robustness check of the consistency of our calibration. Hereafter, we refer to the (first) equilibrium with borrowing fixed costs of $\zeta_1 = 0.6$ as the *pre*-1984 steady-state, and the (second) equilibrium with borrowing fixed costs of $\zeta_2 = 0.4$ as the *post*-1984 steady-state.

Table 3.5.8 reports the quantitative predictions of the model and the effects of financial development on default rates and credit spreads. The model successfully explains the dramatic rise in default rates. The *post*-1984 steady-state is characterized by an average default rate of 1.2%, implying a 300% increase between the two periods. Therefore, financial development accounts for the 64% of the total increase of default rates observed since the early 1980's. This result uniquely stems from the *interplay* between financial development and the stochastic fixed cost of operation. On one hand, the reduction in the fixed cost of borrowing tempers the non-linearities of the value function of the firms[28], making the debt a cheaper source of financing. As a result, efficient firms can finance through debt investment and build a more efficient size; conversely, inefficient firms can disinvest part of their capital, without being penalized as much as before due to the lack of collateral. Then, the reduction of the fixed cost of borrowing increases both the average level and volatility of firms' debt. On the other hand, given the higher collateral value of capital, firms can borrow more debt against the same amount of capital, implying an increase in leverage. Together with a higher

[28] In particular, it decreases the marginal utility cost of increasing debt, where utility is measured in terms of expected discounted feature profits.

Table 3.5.8: Predictions of the Model

Moment		*pre-1984*	*post*-1984	Δ *post*-1984/*pre*-1984
Fixed Borrowing Cost	Model	0.6	0.4	−33%
	Data	-	-	-
Median Debt to Asset Ratio	Model	0.29	0.36	24%
	Data	0.24	0.33	37%
Aggregate Default Rate	Model	0.3%	1.2%	300%
	Data	0.3%	1.7%	467%
Aggregate Credit Spread	Model	75bp	77bp	2bp
	Data	91bp	102bp	11bp
Median Expected Recovery Rate	Model	37%	46%	24%
	Data	–	–	–

Note: this table reports the results of the model in two equilibria. The first one, labeled as "*pre*-1984", denotes the version of the model with high fixed costs of borrowing, $\zeta = 0.6$. The second one, labeled as "*post*-1984", denotes the version of the model with low fixed costs of borrowing, $\zeta = 0.4$. The Median Debt to Asset Ratio is computed as the median value among firms' total amount of debt over the sum of firm's profits and undepreciated capital. The Median Expected Recovery Rate defines the median across firms of the sum of firms' profits and undepreciated capital over the total amount of debt over the states in which the expected probability of default is positive.

volatility of debt, this implies a higher volatility of leverage. Indeed, the leverage of the median firm goes up from 0.29 to 0.36, while its volatility rises from 0.17 to 0.20. Thence, it becomes more likely that firms end up in states of the world where they find optimal to default, pushing up the overall default rate of the economy.

Yet, the idiosyncratic productivity shock alone is not sufficient to imply the default of the firms because of the persistency property of the idiosyncratic productivity shock process and the forward-looking nature of the borrowing constraint. Indeed, the persistent nature of the shock makes it highly predictable. A fortiori since the shock is highly persistent, the intermediary anticipates that a low-efficient firm will keep being inefficient in the next period, and, therefore, curtails the amount of loans for which the firm might find tempting to default. Intuitively, this is the reason at the base of the failure experienced by equilibrium default models in providing default in equilibrium. On the other hand, the rare event (small probability) and unpredictable (i.i.d) nature of the stochastic operational cost provides a modeling expedient for introducing a significant amount of unpredictable uncertainty in the economy, which eventually produces defaults in equilibrium.

On the other side of the picture, Table 3.5.8 reports a 2 bp increase in the credit spread, in line with the empirical evidence. With respect to the level, in both equilibria average credit spreads are around 20 bp *lower* than the real ones, which are around 90 bp. This result is not a surprise. Traditionally, macro-models have been having hard time in provide quantitatively reasonable credit spreads. Indeed, Chen et al. [2008] stress how models which are not able to provide sizable equity premium would never be able to predict the right amount of credit spreads, linking the equity premium puzzle to what they call the credit spread puzzle. In order to overcome these difficulties and match the level of credit spreads, we should add aggregate uncertainty in the model, as in Chen [2010]. In this way, we would add a countercyclical default, countercyclical price of risk and procyclical liquidation values, which, in turn would deliver sizable credit spreads.

Table 8 shows that the model *does* predict the dramatic rise in default rates and the constancy of credit spreads. What is the rationale for this result? The answer to this question is purely quantitative, and stems from the magnitude of three counteracting effects that financial development has on credit spreads. To clarify this point, let us restate Equation (3.3.12) as

$$l_i'(1+r_F) + \underbrace{\zeta(1+r_F)}_{\text{Fixed Cost Channel}} = \mathrm{E}_{H(\chi_i'),x_i'|x_i}\Big[\underbrace{(1-\phi_{D,i})(1+r_i')l_i' + \phi_{D,i}}_{\text{Default Risk Channel}}\ \underbrace{L(k_i',x_i')}_{\text{Insurance Channel}}\Big]$$

When pricing a debt, the credit intermediaries evaluate the fixed cost of issuing a loan (*fixed cost channel*), the probability of default of the firm (*default risk channel*) and the amount of insurance provided by the liquidation value in case of default (*insurance channel*).

As seen before, financial development increases (on average) the probability of endogenous default. Therefore, in absence of any form of insurance in case of default, credit spreads would have to increase, tracking monotonically the rise in default rates observed in the data. Nonetheless, there are two channels through which financial development reduces the credit spreads: the fixed cost of borrowing and the loss given default. First of all, financial development reduces by construction the fixed cost of borrowing, and therefore the interest rate charged on the loan. The impact on the interest rate is stronger the higher is the expected probability of default of the firm, contributing to the reduction of the credit spread. Secondly, financial development increases (on average) the liquidation value. Because of the reduction in the financial frictions in the economy, firms behave more optimally (literally, firms are less constrained in their optimization decisions) and increases their size of operation. As a consequence, firms (on average) produce higher profits and have a larger size. To attach some numbers on these dynamics, in the model firms' median profit and median size increase, respectively, by 21.73% and 9.34% when passing from the *pre*-1984 to the *post*-1984 equilibrium. Both these two components enter the definition of the liquidation value $L(k_i', x_i')$, and increase the insurance component of the credit spread. Indeed, the expected recovery rate hikes up by 24%, from a value of 37% in the first equilibrium to a value of 46% in the second one. Ergo, financial development produces (on average) a dramatic increase in the liquidation value which offsets the increase in the probability of default, producing just a 2 bp increase in the credit spreads in the second equilibrium.

3.5.3 Further Results

In the model, financial development is able to account for a number of trends - other than the rise in default rates - which characterized public firms over the last decades. Namely, the model gives relevant predictions on the number of firms distributing dividends, the way firms decide to smooth these dividends over time, and the level of firms' volatility.

Dividend Payout.

First of all, the way public firms pay dividends has substantially changed over time. As reported by Fama and French [2001], the number of publicly traded non-financial non-utility firms distributing dividends was 66.5% in 1978 compared to only 20% in 1999. The decline in percentage of dividend payers is attributed with equal importance to both a tilt of the publicly traded population towards firms with characteristics of firms that have never payed (low earnings, strong investment and small size), and to a general lower propensity to pay. Our model adds insights to both these channels. Secondly, Leary and Michaely [2011] document a steady and substantial increase in the degree of dividends smoothing over the past century. In the model, financial development account quantitatively for these changes in the dynamics of dividends.

As explained above, although the objective function of the firms is linear, due to the presence of endogenous borrowing constraints (and price schedules) their value functions are strictly concave. Therefore, when deciding the value of dividends to distribute, firms trade off level with volatility. In the model - as in the data - the firms distributing dividend are large firms. In the first equilibrium, credit frictions are tight and firms cannot smooth dividends as they wish. This extra volatility is then compensated by a higher level of dividends. When financial development tempers the credit frictions of the firms, they become better able at insuring dividends from the effects of the persistent idiosyncratic shocks. Hence large firms change their payout policy by increasing the smoothing of dividends while reducing their average level. At the same time, financial development allows for the presence of a higher number of small firms, therefore further reducing (through the distribution) the ratio of dividend payers. As shown in Table 9, financial development makes the number of firms distributing dividends to shrink down by 34%. This mechanism explains the 73% of the decline in the number of firms paying dividends observed in the data.

Meanwhile even the degree at which firms smooth dividends rise substantially. For measuring the degree of dividend smoothing, we follow Lintner [1956] by estimating the following regression

$$\Delta d_{i,t} = d_{i,t} - d_{i,t-1} = \alpha + \beta_1 d_{i,t-1} + \beta_2 y_{i,t} + \epsilon_{i,t}$$

where $d_{i,t}$ denotes the dividend of the i-th firm in time t, while $y_{i,t}$ is the value of the firm's sales. We then estimate the speed of adjustment of dividends by $-\hat{\beta}_1$. When this value equals 0, dividends are perfectly smoothed and follow a random walk. As reported in Table 9, in our model financial development makes the estimated speed of adjustments to decline from 0.43 to 0.22, which implies a substantial increase in the degree of firms' dividend smoothing. These values provided by the model are remarkably close the estimates of Leary and Michaely [2011], which find an estimated speed of adjustment of about 0.3 during the 1960's and 1970's and about 0.2 for the most recent years. Moreover, the theory we propose is corroborated by (one of) the main findings in Leary and Michaely [2011]. After testing several extant explanations of the smoothing motive[29], the authors finds smoothing to be prevalent among firms that appear to have the least constrained access to external capital and highest dividend levels, which have all the characteristics of the large firms described in our model.

Firm Volatility.

Campbell et al. [2001] provide evidence on an upward trend in the volatility of public firms' return, which has more than doubled from the 1960's to the late 1990's. Comin and Mulani [2006] and Comin and Philippon [2005] complement this finding by documenting an increase in idiosyncratic volatility of firm's *real* variables, such as real sales and employment. All these papers conjecture the origin of such trends and suggest that increased competition, R&D innovations, changes in the corporate governance of the firms and the institutionalization of equity ownership could have spurred the volatility of firms.
Here we show that financial development could have been another source of such steep increases of volatility. Actually, in the model the rise in the volatility of firms' sales and returns is the other side of the coin of the evidence reported above on dividends. Indeed, firms achieve a higher degree of dividend smoothing by increasing the volatility of their debt, which in turn spurs the fluctuations in investment, and eventually firms' sales and returns. We compute our measure of volatilities as in Comin and Mulani [2006], as

$$\sigma(x_{i,t}) = \sqrt{\frac{\sum_{\tau=t-4}^{t+5} (x_{i,\tau} - \bar{x}_i)^2}{10}}$$

where $\bar{x}_{i,}$ is the average of the variable $x_{i,t}$ between the periods $t-4$ and $t+5$. In what follows, we compute the volatility of two variables:firms' sales $y_{i,t}$ and firms' cum dividend returns $ret_{i,t} = \frac{V(\omega_{i,t}, x_{i,t}) + d_{i,t}}{V(\omega_{i,t-1}, x_{i,t-1})}$. The bottom of Table 9 shows that, in the model, the median volatility of sales rise from a value of 0.14 to 0.24, and the one of returns from 0.12 to 0.20. As a further robustness check, one of the implications of our theory is that the increase in volatility is pervasive across *all* the quantiles of the size distribution, as observed by Comin and Mulani [2006]. Therefore, financial development causes an increase in the volatility of firms' sales and returns which equals 72% and 67%, respectively.

[29] Theory based on asymmetric information, agency considerations external financing costs and tax planning.

Table 3.5.9: Results on Dividend Payout and Firm Volatility

Variable	*pre*-1984	*post*-1984	Δ *post*-1984/*pre*-1984
	A. Dividend Payout Policy		
N. Firm Paying Dividend	58.2%	38.4%	-34%
Degree Speed of Adjustment	0.43	0.22	-49%
	B. Firm Volatility		
Median Sales Volatility	0.14	0.24	72%
Median Stock Return Volatility	0.12	0.20	67%

Note: this table reports the results of the model in two equilibria. The first one, labeled as "*pre*-1984", denotes the version of the model with high fixed costs of borrowing, $\zeta = 0.6$. The second one, labeled as "*post*-1984", denotes the version of the model with low fixed costs of borrowing, $\zeta = 0.4$. The Degree Speed of Adjustment defines the degree at which firms smooth dividends over time. We take the simulated data of the model, run the regression $\Delta d_{i,t} = d_{i,t} - d_{i,t-1} = \alpha + \beta_1 d_{i,t-1} + \beta_2 y_{i,t} + \epsilon_{i,t}$, and consider $-\hat{\beta}_1$ as the estimate of the speed of adjustment.

3.6 Conclusion

In this paper we document a diverging trend between default rates and credit spreads in the US economy over the last 60 years. On one hand, we find evidence in favour of the presence of one structural break in the unconditional mean of default rates in 1984. This date splits the series of default in two samples with very different characteristics. Indeed, the average corporate default rate rose from an average of 0.3%, during the period 1950-1983, to a value of 1.7% over the period 1984-2012. On the other hand, the average credit spreads barely moved, recording a 11 basis point increase. We run a battery of tests to show that this movement in credit spreads is statistically insignificant. Therefore, over the last three decades, default rates experienced a 467% increase, while credit spreads kept constant. Hence, nowadays corporate bankruptcies are more and more frequent than thirty years ago, but this came at no effect on the average borrowing cost.

We present a dynamic equilibrium model with heterogeneous firms where the development of credit markets and limited enforceability of debt contracts can be accounted for the diverging trend between rising default rates and constant credit spreads. We model the development of credit markets through an exogenous reduction of fixed costs of borrowing, as a reduced form for the development of the U.S. financial system during the 1970's and 1980's. The predictions of the model are quantitatively appealing. Financial development accounts for the 64% of the rise in the default rates, which is accompanied by an increase in credit spreads of just 2 basis points. Indeed, the reduction in the fixed borrowing costs make debt cheaper:

Firms can access larger loans to invest more in capital and grow up in size. At the same time, firms that become inefficient can disinvest without being as penalized as before in their interest rates, due to the lack of collateral. So, the volatility of investment goes up, just because debt becomes more volatile too. Hence, financial development increases the level of debt, its volatility and makes eventually default more likely. On the other hand, credit spreads barely move because the insurance channel due to the financial development prevails on the default risk channel. Indeed, if on one side the cut in the borrowing fixed costs increases (on average) the endogenous probability of default in the economy, on the other side it reduces (on average) the wedge between the actual firms' optimal choices and the frictionless ones. As a consequence, financial development increases both the median size of capital by 9.34% and median profits by 21.73%. The upsurge in the expected liquidation value of the firms offsets the dramatic rise in default rates, leaving the credit spreads unchanged.
Furthermore, we show that in the model financial development can account for a number of trends - other than the increase in default rates - that characterized public firms in the last thirty years. First, we show that the reduction of the fixed credit costs changes firms' optimal decisions of dividend payout. Since firms are now better able to smooth dividends over time, they can trade off this reduction in volatility with a decrease in the level of dividends. As a result of financial development, the measure of firms distributing dividends shrinks down by 33%. This number accounts for the 73% of the decline documented for the U.S. by Fama and French [2001]. Furthermore, in the model the median degree of dividend smoothing increases of a magnitude which is remarkably close to the values estimated by Leary and Michaely [2011]. Second, we study the volatility of firms' returns and sales. Indeed, Campbell et al. [2001], Comin and Mulani [2006], Comin and Philippon [2005] show the presence of a secular upward trend in the volatilities of firms. We suggest that this empirical evidence can be (at least partially) accounted for by financial development. Indeed, the model is able to reproduce a 72% in the volatility of sales and a 67% for firms' returns.

Appendix

3.A Appendix: The Role of Financial Development

Assumptions 1. Let us assume the following:

1. $\delta = 1$: full depreciation. This assumption implies that capital is not anymore a state.
2. $\psi = 1$: full clearance loss which implies no liquidation value.
3. $x_i = \bar{x}_i$: each firm initially experiences (is endowed with) a different idiosyncratic shock, henceforth constant. For example, firms are born of a particular type, where the type captures different levels of productivity.
4. Firms do not suffer the stochastic fixed cost of operation.
5. Firms cannot issue equity.
6. Entrants are endowed with no debt and no capital.
7. The subjective time discounting parameter β equals $\frac{1}{1+r_F}$.

Under these assumptions, operating profits and the firm net-worth reduces to:

$$\pi_i = x_i k_i^{\alpha}$$
$$w_i = \pi_i - (1 + r_i) l_i$$

Let us now characterize the equilibrium properties of this economy. Since there is no uncertainty in the model, in equilibrium the firms optimal policies are constant, $(\phi_D^*, k'^*, l'^*, d^*)$. In principle there are two putative equilibria where firms choose with probability one to default ($\phi_D^* = 1$) or not to default ($\phi_D^* = 0$). On one hand, we can show that the defaulting equilibrium is not an equilibrium for the firm. The proof is trivial and is made by contradiction.

Proof. Let us assume that a firm defaults in equilibrium; this means that it exits without distributing any dividend. This strategy violates the profit maximizing condition. We can find a feasible strategy that delivers a higher payoff. In particular not defaulting, producing at the first best level of capital each period, distributing the rest in dividend (without using debt) is a feasible strategy with constant policies which provides a higher payoff. □

On the other hand we can show that the No-Defaulting equilibrium holds only for a bounded set of loans. Despite the proof is more involved, the intuition is straightforward. An increase in the accorded level of debt increases the incentive for the firm to deviate from the equilibrium policy of not defaulting, distributing a big dividend in the current period and defaulting the period later. In particular in what follows we will show that there exists a threshold value of debt, $l_{D,i}^*$ above which firms will find optimal to deviate from the no default equilibrium strategy. Let us prove it.

Proof. The proof articulates as follows:

1. Proposition 3.A.1 states the zero-profit condition of the intermediary under Assumptions 3.A
2. We plug the result of Proposition 3.A.1 in the problem of the incumbents and we find the optimal policy of capital
3. We define the incentive compatibility constraint of the firm given the above policy functions
4. We define from the incentive compatibility constraint of the firm the debt-limit

Proposition 3.A.1. *Under Assumptions 3.A, the equilibrium zero profit condition* $(l_i^*, r_i^*) \in \Omega(l_i^*, \bar{x}_i; \lambda)$ *can be rewritten as:*

$$(1 + r_i^*)l_i^* = (1 + r^*)l_i^* + (1 + r^*)\zeta \tag{3.A.1}$$

Proof. The credit intermediary zero profit condition reduces to:

$$\begin{aligned}
l_i' + \zeta &= \frac{\mathrm{E}_{x'|x}\left[(1 - \phi_{D,i})(1 + r_i)l_i' + \phi_{D,i}L(k')\right]}{1 + r} \\
l_i' + \zeta &= \frac{(1 - \phi_{D,i})(1 + r_i')l_i' + \phi_{D,i}L(k')}{(1 + r)} \\
l_i' + \zeta &= \frac{(1 - \phi_{D,i})(1 + r_i')l_i'}{(1 + r)}
\end{aligned}$$

The first step is a result of the absence of idiosyncratic uncertainty while the second one comes from the absence of liquidation value. Since we showed before that the firm does not default in equilibrium, the equilibrium level of debt obeys:

$$l_i^* + \zeta = \frac{(1 + r_i^*)l_i^*}{(1 + r^*)} \tag{3.A.2}$$

which concludes the proof.

□

Hence the firms in equilibrium can borrow at the risk free rate, corrected for the fixed cost of borrowing. This result stems from the fact that firms do not default in equilibrium.
However, this result does not imply that firms can borrow as much as they want. This point is made clear following the proof.
In order to derive further insights on the firms optimal behaviors let's analyze the problem of the incumbents. Under Assumptions 3.A it reduces to:

$$\begin{aligned}
V_i &= \max_{\{k_i', l_i'\}} d_i + \frac{1}{1 + r} V_i(x_i'; \lambda') \\
\text{s.t} \quad d_i &= \bar{x}_i k_i^{\alpha} - k_i' + l_i' - (1 + r_i)l_i \\
&\left(l_i', r_i'\right) \in \Omega(l_i', \lambda)
\end{aligned}$$

Substituting the zero profit condition (3.A.1):

$$
\begin{aligned}
V = \max_{\{k_i', l_i'\}} \; & d_i + \frac{1}{1+r} V_i(x_i'; \lambda') \\
\text{s.t} \quad & d_i = \bar{x}_i k_i^{\alpha} - k_i' + l_i' - (1+r) l_i - (1+r)\zeta
\end{aligned}
$$

The first order necessary conditions reads:

$$
\begin{aligned}
rCl1 &= \frac{\alpha \bar{x}_i (k_i')^{\alpha-1}}{1+r} && (k_i) \\
1 &= \frac{1+r}{1+r} && (b_i)
\end{aligned}
$$

from which we obtain:

$$
rCl1 + r = \alpha \bar{x}_i (k_i')^{\alpha-1} \qquad (k_i)
$$

Hence in equilibrium firms can equal their marginal product to the risk free rate, choosing the first best level of capital:

$$
k^* = k_{fb,i} = \left(\frac{\alpha \bar{x}_i}{1+r} \right)^{\frac{1}{1-\alpha}} \tag{3.A.3}
$$

Notice that the firms still suffer a dead-weight loss due to the fix-cost of borrowing ζ, but this burden does not affect the optimal choice of capital which is taken at the margin. At this point we can define the firm incentive compatibility constraint as the feasible set of policies strategies for which the firm does not want to default. In equilibrium:

$$
\Phi(\omega_i^*, \bar{x}_i; \lambda) = \left\{ (d_i^*, l_i^*, k_i^*) \in \mathbb{R}^2 \times \mathbb{R}_+ : \omega_i^* + d_i^* + l_i^* - k_i^* \geq 0 \right\} \tag{3.A.4}
$$

Given the monotonicity properties of the firms value function there exists a firm specific debt limit $l_{D,i}$, such that any accorded level of debt higher than this debt limit, will provide an incentive for the firm to deviate and default. This interpretation rationalizes the label endogenous borrowing constraint.
The debt limit is defined as the level of debt for which the optimal policy functions deliver a zero-net worth:

$$
\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} + l_{D,i} - (1+r) l_{D,i} - (1+r)\zeta = 0
$$

which simplifies to:

$$
\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} - r l_{D,i} - (1+r)\zeta = 0 \tag{3.A.5}
$$

Proposition 3.A.2. *A no defaulting equilibrium strategy for a firm i is sustained for level of debt $l_i^* \in [0, l_{D,i}]$, where $l_{D,i}$ represent the equilibrium firm-specific debt limit and it is defined as:*

$$
l_{D,i} = \frac{1+r-\alpha}{r\alpha} k_{fb,i} - \frac{(1+r)}{r} \zeta \tag{3.A.6}
$$

Proof. Substituting (3.A.3)

$$
\begin{aligned}
l_{D,i} &= \frac{\bar{x}_i k_{fb,i}^{\alpha} - k_{fb,i} - (1+r)\zeta}{r} \\
&= \frac{\bar{x}_i \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{\alpha}{\alpha-1}} - \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} - (1+r)\zeta}{r} \\
&= \frac{\bar{x}_i^{-\frac{1}{\alpha-1}} \left(\frac{1+r}{\alpha}\right)^{\frac{1}{\alpha-1}+1} - \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} - (1+r)\zeta}{r} \\
&= \frac{\left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} \left[\frac{1+r}{\alpha} - 1\right] - (1+r)\zeta}{r} \\
&= \frac{\left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}} \left[\frac{1+r-\alpha}{\alpha}\right] - (1+r)\zeta}{r} \\
&= \frac{\left[\frac{1+r-\alpha}{\alpha}\right] \left(\frac{1+r}{\alpha \bar{x}_i}\right)^{\frac{1}{\alpha-1}}}{r} - \frac{(1+r)\zeta}{r} \\
&= \frac{1+r-\alpha}{r\alpha} k_{fb,i} - \frac{1+r}{r}\zeta
\end{aligned}
$$

□

Now, we can easily derive the following derivatives:

$$
\begin{aligned}
rCl\frac{\partial l_{D,i}}{\partial \zeta} &= -\frac{1+r_i}{r_i} < 0 \\
\frac{\partial l_{D,i}}{\partial k_{fb,i}} &= \frac{1+r_i-\alpha}{r\alpha} > 0
\end{aligned}
$$

reported in Proposition 3.4.3. First, the debt-limit is increasing in the level of financial development. The lower the level of financial development (the higher is ζ), the lower is the level of debt for which the firm is indifferent whether to default or not. Second, the debt-limit is increasing in the optimal choice of capital, which, in this context, is function uniquely of the original idiosyncratic shock. The higher is the idiosyncratic shock (quality of production), the higher is the debt limit, i.e. the threshold of debt for which a firm is indifferent whether to default or not.

□

Let us now define the leverage at the debt limit as:

$$
\frac{l_{D,i}}{k_{fb,i}} = \frac{1+r_i-\alpha}{r_i\alpha} - \frac{1+r_i}{r_i}\frac{\zeta}{k_{fb,i}} \tag{3.A.7}
$$

From which we can easily derive the following derivatives:

$$
\begin{aligned}
rCl\frac{\partial \frac{l_{D,i}}{k_{fb,i}}}{\partial k_i^{fb}} &= \frac{1+r_i}{r_i}\frac{\zeta}{(k_{fb,i})^2} > 0 \\
\frac{\partial \frac{l_{D,i}}{k_{fb,i}}}{\partial \zeta} &= -\frac{1+r_i}{r_i}\frac{1}{k_{fb,i}} < 0
\end{aligned}
$$

reported in Proposition 3.4.4. First, the leverage is strictly increasing in the amount of capital. The higher is the productivity of a firm, the higher is the optimal level of capital, the higher is the equilibrium leverage. Second, the leverage is strictly increasing in the level of financial development. The more developed is the credit intermediation, the lower is ζ, the higher is the equilibrium leverage of the firms.

3.B Appendix: Data

3.B.1 Firm Characteristics

Data on firms characteristics are taken from Compustat, fundamental annual data from 1950 to 2006. Compustat includes public firms listed on the three US exchanges, NYSE, AMEX, and Nasdaq, with a non-foreign incorporation code.

Following Covas and Den Haan [2011], we exclude: 1) American Depository Receipts (ADRs) - securities created by U.S. banks to permit a U.S.-based trading of stocks listed on foreign exchanges; 2) financial firms (SIC classification between 6000 and 6999); 3) utilities (SIC classification between 4900 and 4949); 3) firms involved in major mergers (Compustat footnote code AB[30]); 4) firms with missing value for the book value of assets.

Entrants are defined as firms which are showing up on Compustat for the first time.

The assets, ($a \equiv k_{it} + xk_{it}^{\alpha}$), in the model, are computed as the book value of assets (Compustat data item 6 - mnemonic AT).

The total debt, (b_{it}) in the model, is computed as long-term debt (item 9 , mnemonic) plus debt in current liabilities (item 34, mnemonic), since there is no distinction among the two in the present model.

3.B.2 Default Rate

Data on corporate default rates are taken from Giesecke et al. [2011].

3.B.3 Credit Spreads

Data on credit spreads are taken from the St Louis FRED. Credit spreads are computed using the monthly seasonally not-adjusted Moody's Seasoned Corporate Bond Yield, from 1950 to 2012. The series follows an investment bond that acts as an index of the performance of all bonds given a specific rating by Moody's Investment Firm. Annual series are constructed by averaging the monthly percent bond yields. The spread is then computed as the difference of the natural logarithm of BAA and AAA bond yields.

3.B.4 Ex-post Real Risk Free Interest Rate

The ex-post real risk free interest rates are computed using data on 1) the Treasury Constant Maturity Rate bill, from the St Louis FRED, and 2) the Personal Consumption Expenditures (PCE) Chain-type Price Index, from the Bureau of Economic Analysis.

[30] Compustat assigns a footnote AB to total sales if sales increase by more than 50 percent in response to a merger or an asset acquisition.

The ex-post real risk free interest rate is computed as the difference between the three-month Treasury bill rate minus the realized inflation in the subsequent quarter.
We use the three-month Treasury Constant Maturity Rate, at monthly frequency. We then build annual data averaging (equal weights) the monthly rates.
For the inflation, we use the seasonally adjusted quarterly rate of the Personal Consumption Expenditures (PCE) Chain-type Price Index. The annualized growth for PCE deflator is computed by taking 400 times the first differences of the natural logs of the PCE deflator. The series of ex-post real interest rate so constructed goes from 1950 to 1985.

3.B.5 GDP growth

We take the data on real GDP from St Louis FRED. We compute the GDP growth from 1949 until 2011 as the log difference of the raw GDP data. As a robustness check, we also use HP-filtered data ($\lambda = 6.25$).

3.B.6 Stock Returns Volatility

We take daily data on stock returns from CRSP. We compute the annual volatility from 1949 until 2011 by computing the standard deviation of returns within a year. Since the measure is computed over non-overlapping spans of time, the measurement errors are uncorrelated and do not bias the estimates of volatility.

3.C Appendix: Computational Algorithm

The computation adopts the discrete choice method. Grids on bond and capital consist of 200 grid points. The computational algorithm articulates as follows:

1. It starts with the guess of: 1) the continuation value function of the incumbents; 2) the default policy function of the incumbents; 3) the debt schedules of the incumbents. In line with Arellano et al. [2012] the initial guess of the debt schedules is the risk free interest rate.

2. It iterates over the continuation value function of the incumbents in the fixed point algorithm till convergence.

3. The implied continuation value function is used for updating, through the optimal default decision rule (3.3.5), the default policy function. Clearly the convergence of the value function implies the convergence of the default policy functions, but not *viceversa*.

4. The implied default policy functions are used to update the endogenous probability of default, which in turns is used for updating the feasible correspondence Ω-set and, therefore, the debt schedules (3.3.12).

5. Points 2, 3, 4 are iterated till convergence of the debt schedules.

Technical Details:

- The levels of tolerance for the convergence of the value function and of the debt schedules are set to 1e-6.
- The grids are controlled not to be binding in equilibrium.
- The statistics reported in Table 3.5.8 are obtained using the (ergodic) distribution at period T=1000, obtained simulating 15000 firms over 1000 periods, .
- The Tauchen [1986] algorithm truncates the $\pm\inf$ values of the support of the normal distribution at $\pm 20\sigma_{log(x)}$.

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